Partial Mechanism Design & Incomplete-Information Industrial Organization

EC'23 Tutorial

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Motivation

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#2. How should a social planner optimally redistribute? (with a private market)

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In these examples, the mechanism designer can design only part of the market.

Mechanism design with equilibrium effects

#1. Consumers freely participate in designed market; allocations are realized.

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Question: How should the mechanism designer allocate the good in **#1**?

There are equilibrium effects: allocations in **#1** affect outcomes in **#2**—and vice versa.

Goals of this tutorial

#1. Introduce a relatively new and rapidly growing research program.

- Recent revived interest in applying large-market mechanism design to applied problems:

  Condorelli (2013); Dworczak ⓡ al. (2021); Akbarpour ⓡ al. (2021); Kang (2022); Akbarpour ⓡ al. (2022); Pai and Strack (2022)...

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- Adapt some classic tools from mechanism design.

#3. Conclude with some open questions.

Framework

Model: consumers

There is a unit mass of risk-neutral consumers with unit demand + quasilinear utility.

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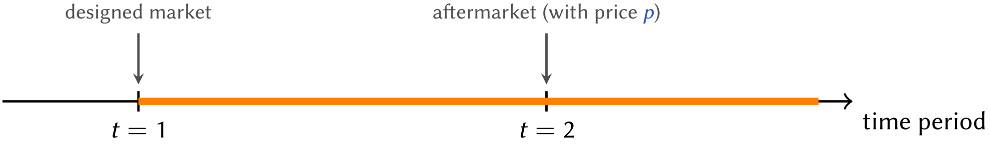

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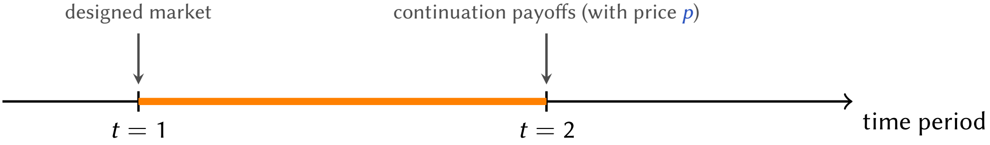

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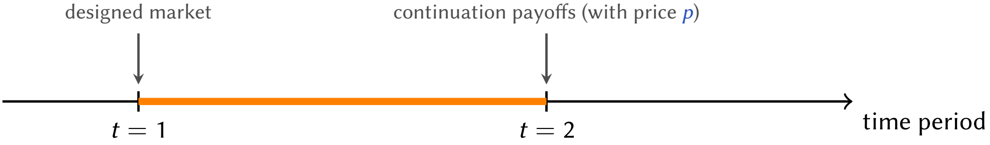

Continuation payoffs (with price p):

$$\begin{cases} v_0(p;\theta) & \text{if \underline{not} allocated the good in the designed market,} \\ v_1(p;\theta) & \text{if allocated the good in the designed market.} \end{cases}$$

Model: mechanism designer

There is a mechanism designer who chooses a direct mechanism (x, t), consisting of:

- an allocation function $x : [\underline{\theta}, \overline{\theta}] \to [0, 1]$, where $x(\theta)$ = prob. that consumer receives good; and
- a payment function $t : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}$, where $t(\theta)$ = expected payment that consumer makes.

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Given p, the payoff of the mechanism designer is, for some increasing function $\Pi_1 : \mathbb{R} \to \mathbb{R}$ and some function $\Pi_0 : \mathbb{R}_+ \to \mathbb{R}$,

$$\Pi(x; p) = \Pi_1\left(\int_{\underline{\theta}}^{\overline{\theta}} \psi(\theta; p)x(\theta)\, \mathrm{d}F(\theta)\right) + \Pi_0(p).$$

Key assumption (A): Π is an affine functional of x (up to increasing transformation).

Model: aftermarket

The price p in the aftermarket depends on the mechanism (x, t) through

$$\phi(p) = P(x), \quad \text{for some function } \phi : \mathbb{R}_+ \to \mathbb{R}.$$

Denote the laissez-faire price by p_0, so that $\phi(p_0) = P(0)$.

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- Allocation x changes residual demand and residual supply in the aftermarket.
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Key assumption (B): $P(x)$ is an affine functional of x.

Key trade-off: direct effect versus indirect effect

$$\Pi(x;p) - \Pi(0;p_0) = \underbrace{\Pi(x;p) - \Pi(0;p)}_{\textbf{direct effect}\ (\text{price} = p)} + \underbrace{\Pi(0;p) - \Pi(0;p_0)}_{\textbf{indirect effect}\ (\text{price}\ p_0 \to p)} .$$

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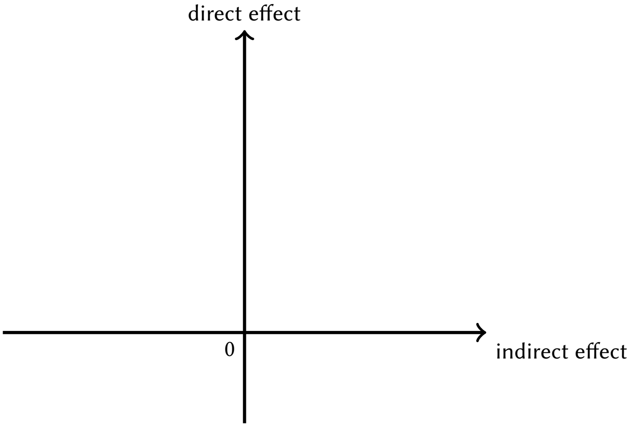

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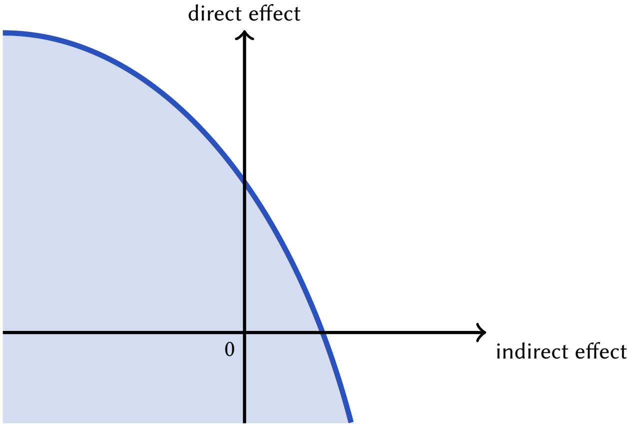

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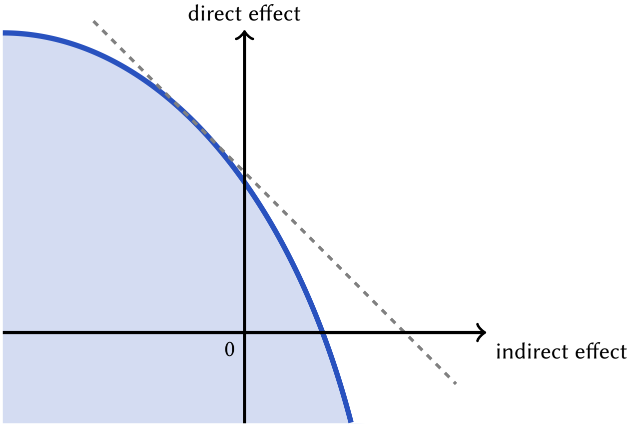

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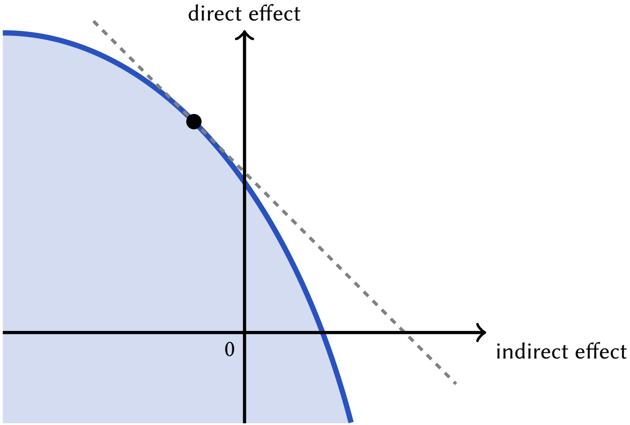

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Analysis

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To answer this question, we proceed in two steps:

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2. Choose mechanism (x, t) that induces the equilibrium price p in aftermarket in (1).

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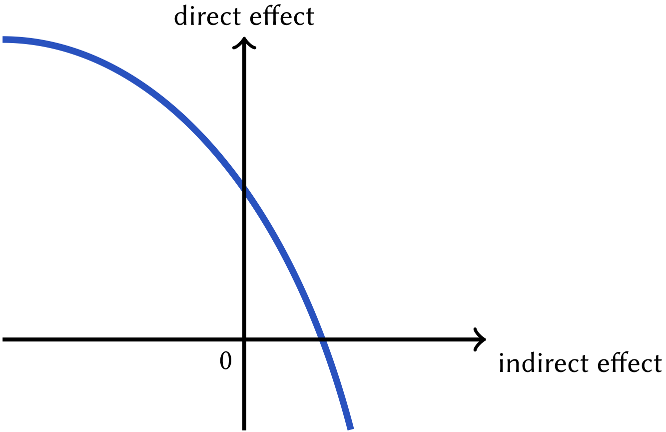
   For the chosen value of indirect effect, maximize the direct effect $\Pi(x; p) - \Pi(0; p)$.

Solving the mechanism design problem



Choose mechanism that maximizes direct effect at any given value of the indirect effect.

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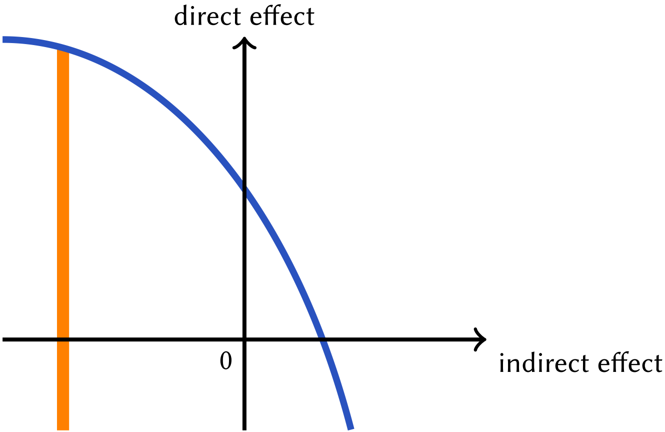

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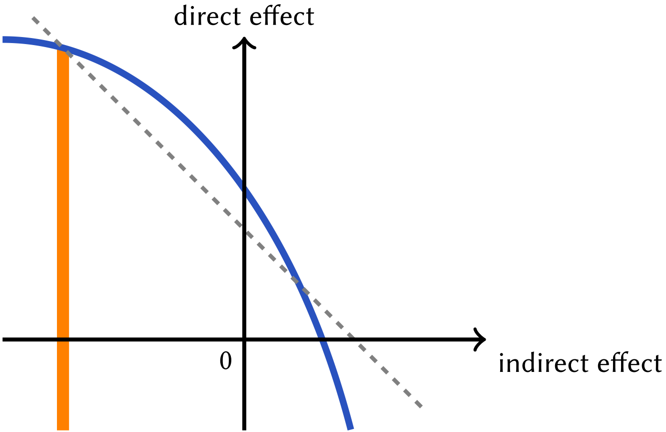

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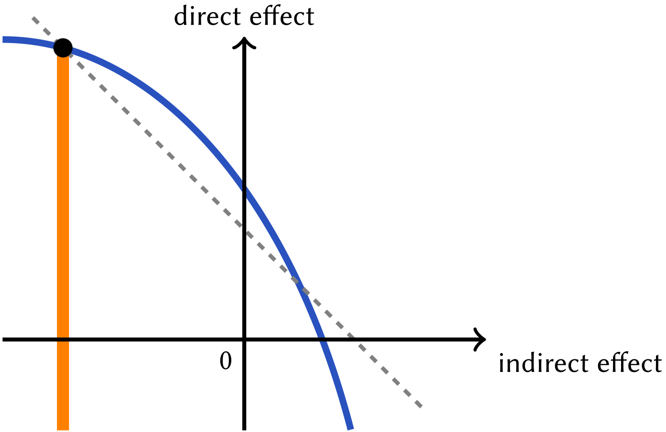

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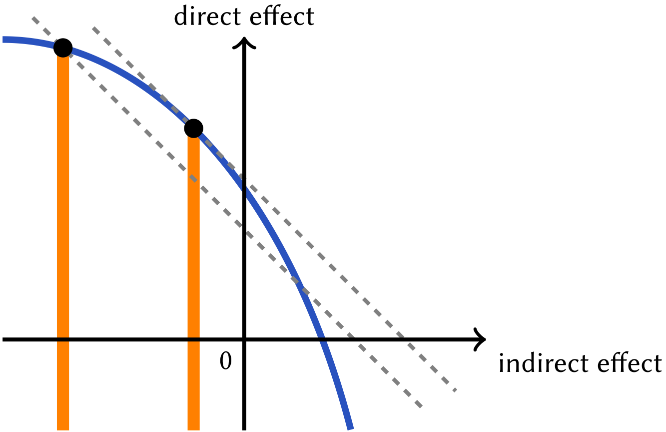

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Constrained mechanism design

For each price p: $$\max_{(x,t)} \underbrace{\Pi(x;p) - \Pi(0;p)}_{\text{direct effect}}$$

$$\text{s.t. } P(x) = \phi(p) \iff \underbrace{\Pi(0;p) - \Pi(0;p_0)}_{\text{indirect effect}} = \Delta\Pi_I$$

and (x, t) satisfies incentive constraints (IC) and (IR).

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- **Incentive compatibility**: consumers report their types truthfully, *i.e.*,

$$\theta \in \arg\max_{\theta' \in [\underline{\theta}, \overline{\theta}]} \left\{ v_1(p;\theta) \cdot x(\theta') + v_0(p;\theta) \cdot \left[1 - x(\theta')\right] - t(\theta') \right\} \quad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]. \qquad \text{(IC)}$$

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- **Individual rationality**: consumers participate in the designed market voluntarily, *i.e.*,

$$v_1(p;\theta) \cdot x(\theta) + v_0(p;\theta) \cdot [1 - x(\theta)] - t(\theta) \geq v_0(p;\theta) \quad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]. \tag{IR}$$

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This is equivalent to:

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Define the **effective type** $\eta(\theta; p) := v_1(p;\theta) - v_0(p;\theta)$.

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Individual rationality (IR) with an aftermarket

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$\therefore$ WLOG, the designer considers mechanisms that elicit only information about η.

Myerson's lemma

Myerson's lemma holds up to a change of variables from θ to $\eta(\theta; p)$:

Lemma 1. For any given price p, let

$$\underline{\eta} = \min_{\theta \in [\underline{\theta}, \overline{\theta}]} \eta(\theta; p) \quad \text{and} \quad \overline{\eta} = \max_{\theta \in [\underline{\theta}, \overline{\theta}]} \eta(\theta; p).$$

Then any mechanism (x, t) satisfies (IC) and (IR) only if there exist a non-decreasing function $\hat{x} : [\underline{\eta}, \overline{\eta}] \to [0, 1]$ and a function $\hat{t} : [\underline{\eta}, \overline{\eta}] \to \mathbb{R}$ such that

1. $x(\theta) = \hat{x}(\eta(\theta; p))$ almost everywhere; and
2. $t(\theta) = \hat{t}(\eta(\theta; p))$ almost everywhere, such that

$$\eta \cdot \hat{x}(\eta) - \hat{t}(\eta) = \underline{\eta} \cdot \hat{x}(\underline{\eta}) - \hat{t}(\underline{\eta}) + \int_{\underline{\eta}}^{\eta} \hat{x}(s)\, \mathrm{d}s \quad \text{for all } \eta \in [\underline{\eta}, \overline{\eta}] \quad \text{and} \quad \underline{\eta} \cdot \hat{x}(\underline{\eta}) - \hat{t}(\underline{\eta}) \geq 0.$$

Optimal mechanism

Main theorem. There exists an optimal mechanism (x^*, t^*) that is a menu of at most two prices; that is, $\operatorname{im} x^* \subseteq \{0, \pi, 1\}$ for some $0 < \pi < 1$.

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This provides a new justification for rationing: to trade off direct and indirect effects.

Proof idea of main theorem

Main theorem. There exists an optimal mechanism (x^*, t^*) that is a menu of at most two prices; that is, $\operatorname{im} x^* \subseteq \{0, \pi, 1\}$ for some $0 < \pi < 1$.

The designer's problem can be expressed as:

$$\max_{\hat{x}} \int_{\underline{\eta}}^{\overline{\eta}} \mathbf{E}[\psi(\theta; p) \mid \eta] \hat{x}(\eta)\, \mathrm{d}G(\eta)$$

$$\text{s.t.} \begin{cases} \hat{x} : [\underline{\eta}, \overline{\eta}] \to [0, 1] \text{ is non-decreasing,} \\ \phi(p) = P(\hat{x}). \end{cases}$$

This is an infinite-dimensional linear program: objective and constraint are **affine** in $\hat{x}$.

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Any extreme point x^* of the feasible region satisfies $\operatorname{im} x^* \subseteq \{0, \pi, 1\}$ for some $0 < \pi < 1$.

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Zi Yang Kang

Harvard University

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Application #1: Vertical Contracting

inspired by Kang and Muir (2022)

Motivation

- Many dominant firms do not necessarily sell a final good directly to consumers; instead, they sell an input good to suppliers, who use it to produce a final good.
  - For example, Amazon sells distribution services to merchants, who then sell to downstream consumers; Google sells ads to third-party sellers, who use ads to make sale.

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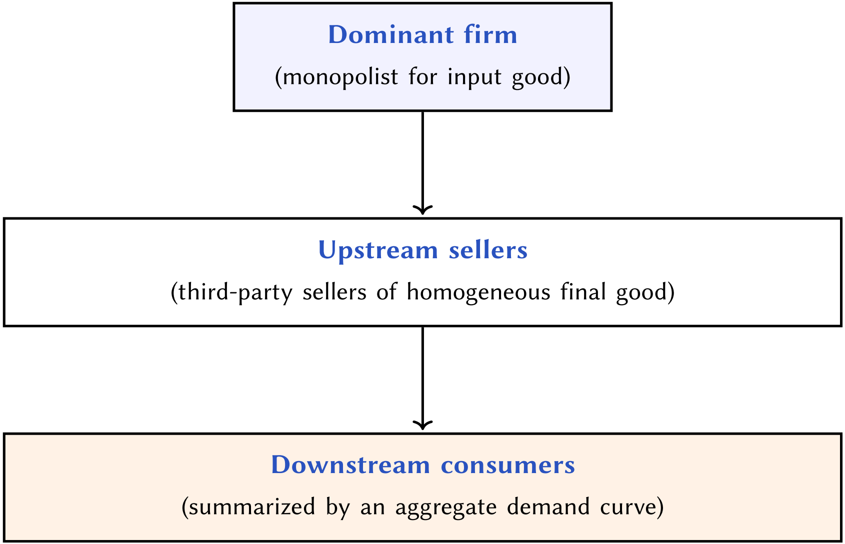
Question: How does the dominant firm optimally contract with third-party sellers?

Overview of model



Mapping from framework: upstream sellers

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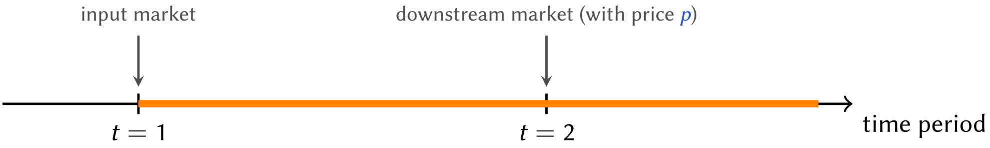

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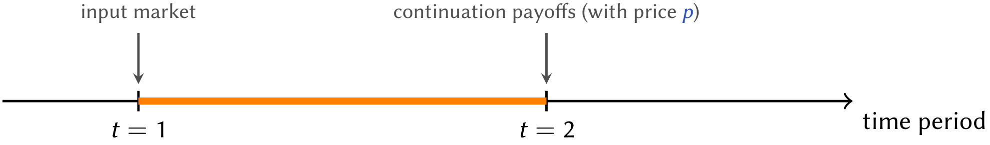

Mapping from framework: upstream sellers

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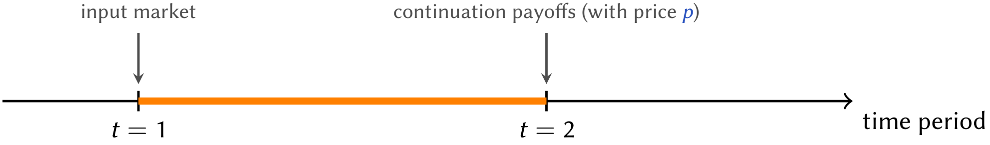

Continuation payoffs (with price p per unit of final good):

$$\begin{cases} v_0(p;\theta) = 0 & \text{if } \underline{\text{not}} \text{ allocated input,} \\ v_1(p;\theta) = \theta p & \text{if allocated input.} \end{cases}$$

Mapping from framework: mechanism designer

There is a dominant firm who chooses a direct mechanism (x, t), consisting of:

- an allocation function $x : [\underline{\theta}, \overline{\theta}] \to [0, 1]$, where $x(\theta)$ = prob. that seller receives input; and
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Given p, the payoff of the dominant firm is

$$\Pi(x; p) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta - \frac{1 - F(\theta)}{f(\theta)}\right] px(\theta)\, \mathrm{d}F(\theta).$$

Key assumption (A): Π is an affine functional of x.

Mapping from framework: downstream market

The price p in the downstream market depends on the mechanism (x, t) through

$$D(p) = Q_0 + \int_{\underline{\theta}}^{\overline{\theta}} \theta x(\theta)\, \mathrm{d}F(\theta).$$

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- Demand for final good is inelastic and captured by downward sloping demand curve $D(p)$.
- More productive sellers have higher WTP for input, but drive down downstream price more.

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- Demand for final good is inelastic and captured by downward sloping demand curve $D(p)$.
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Key assumption (B): total supply of final good (RHS) is an affine functional of x.

Optimal mechanism

Main theorem. There exists an optimal mechanism (x^*, t^*) that is a menu of at most two prices; that is, $\operatorname{im} x^* \subseteq \{0, \pi, 1\}$ for some $0 < \pi < 1$.

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Exercise: rationing is not optimal if $\frac{1-F(\theta)}{\theta f(\theta)}$ is decreasing in θ.

Application #2: Public Option

based on Kang (2023)

Motivation

- Governments often redistribute by providing public alternatives to goods sold in private markets, many of which are allocated at prices below market-clearing levels.
  - For example, public housing programs allow eligible individuals to rent affordable housing units at lower prices relative to private apartments of similar quality.
  - This results in excess demand: in the United States, 1.6 million households were on a public housing waitlist in 2012 (Collinson et al., 2016).

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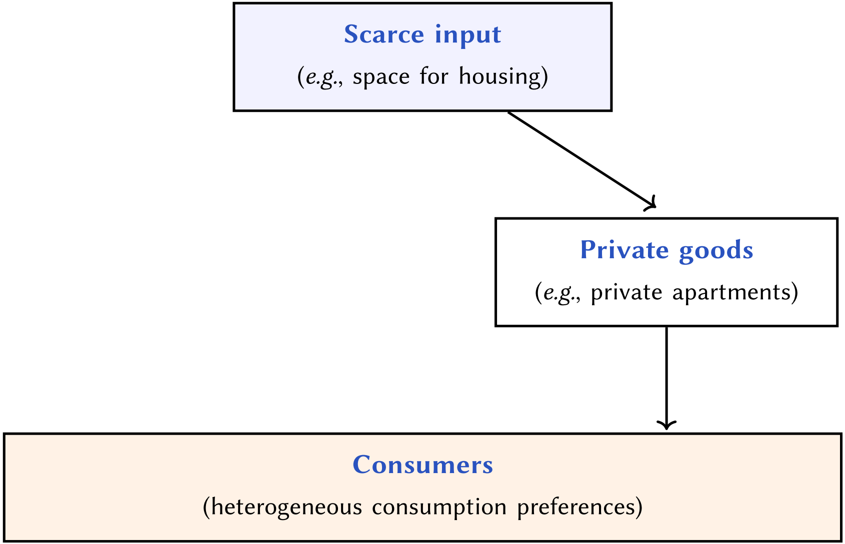
Question: Can rationing be optimal in the long run and, if so, why?

Overview of model



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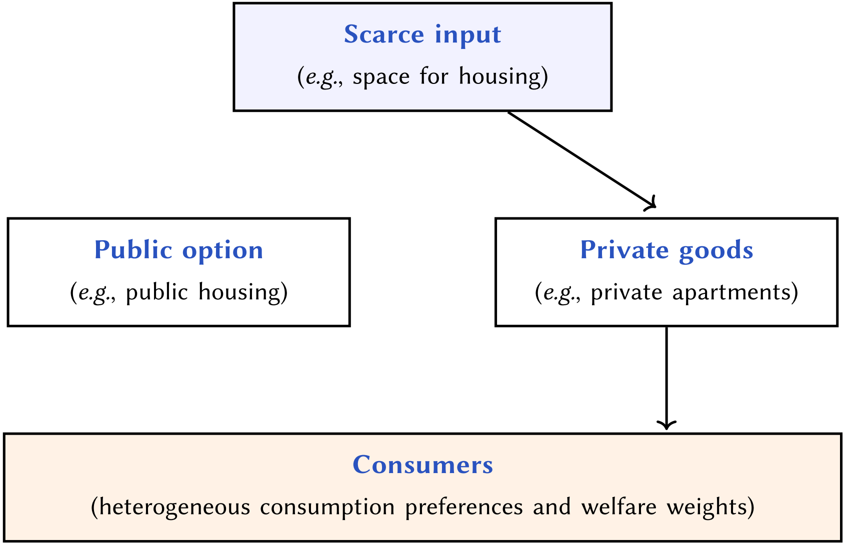

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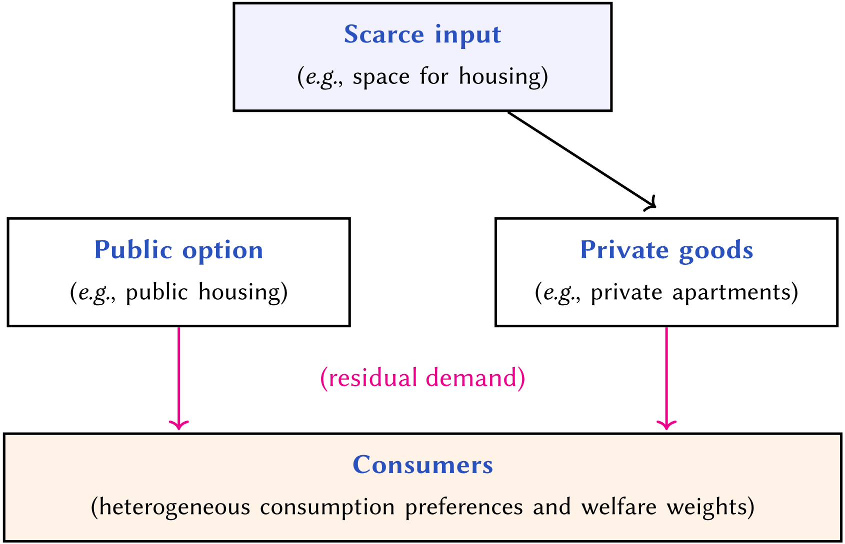

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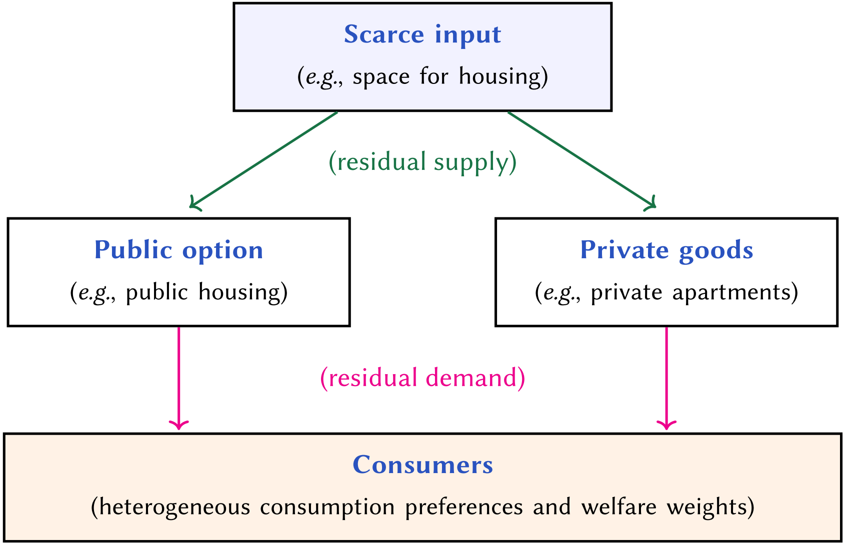

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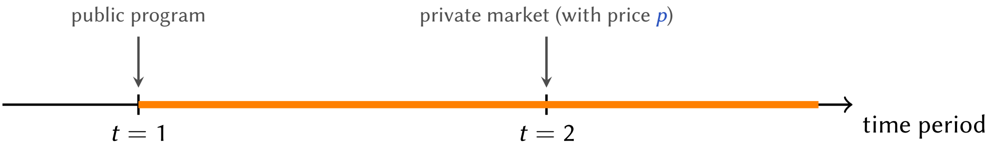

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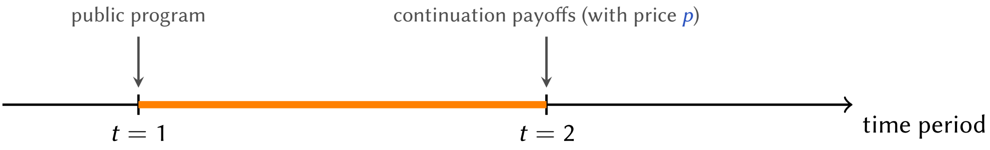

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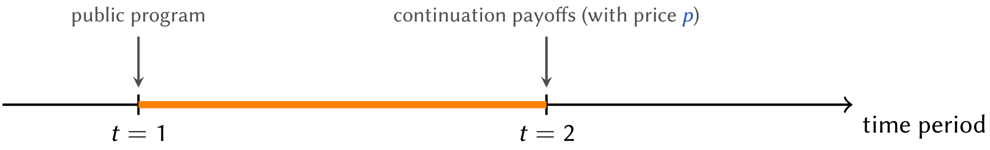

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Mapping from framework: private market

The price p in the private market depends on the mechanism (x, t) through

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Idea:

- Space for housing is scarce; its supply is captured by upward sloping supply curve $S(p)$.
- Public housing units potentially crowd out private apartments and drive up price of space.

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> **Key assumption (B):** total demand for space (RHS) is an affine functional of x.

Implications of incentive constraints

Proposition 1. For any incentive-compatible mechanism (x, t), the probability of receiving the public option is quasiconcave in θ.

- **Proof**: allocation probability must be increasing in $\eta(\theta; p) = u(q_{\text{public}}, \theta) - v_0(p; \theta)$.

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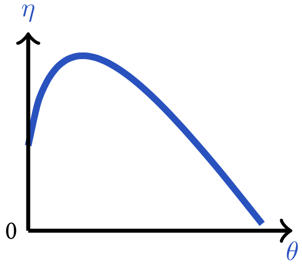

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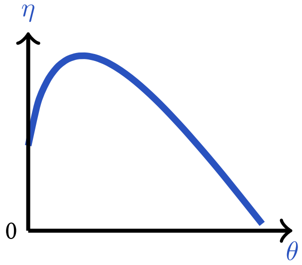

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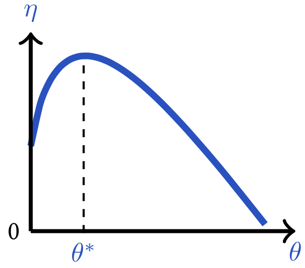

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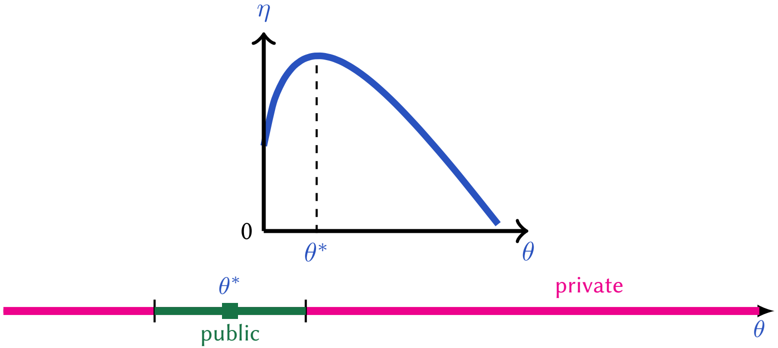

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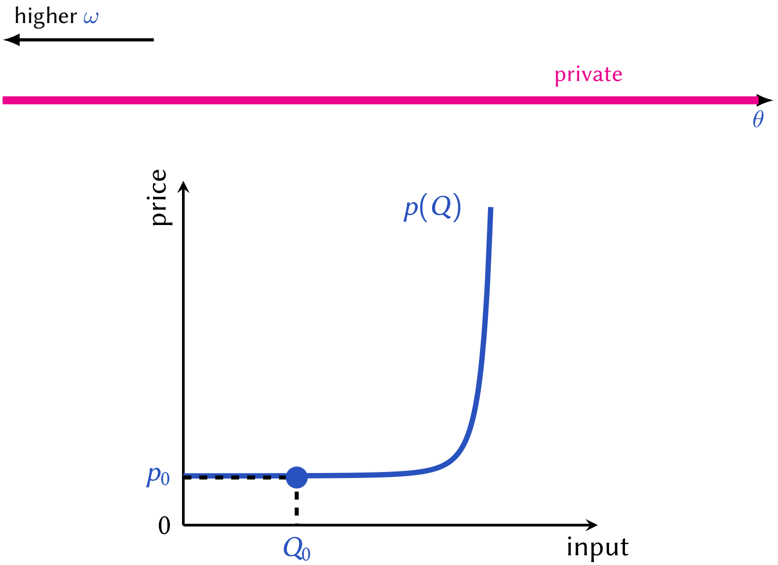
Thus rationing can be optimal in the long run to trade off direct and indirect effects.

- **Direct effect:** $\Pi(x; p) - \Pi(0; p)$ measures the value due to screening.
- **Indirect effect:** $\Pi(0; p) - \Pi(0; p_0)$ measures the value due to pecuniary externalities.

Intuition for rationing



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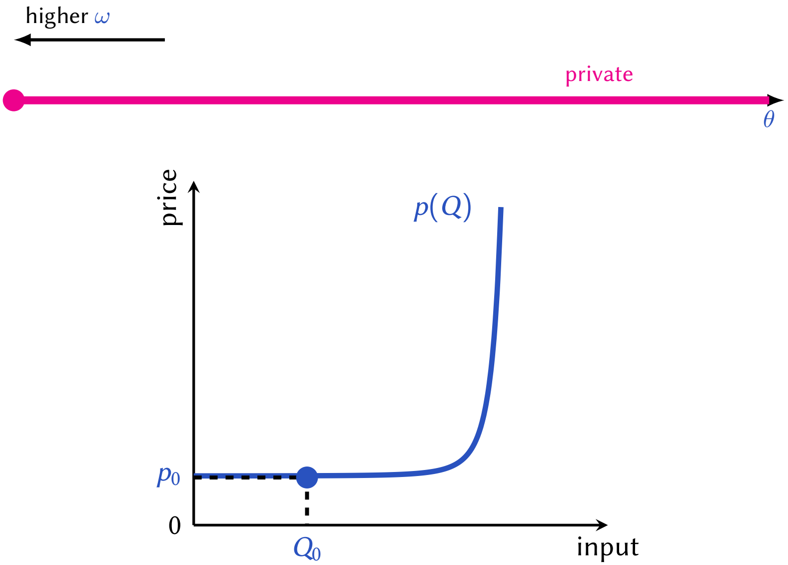

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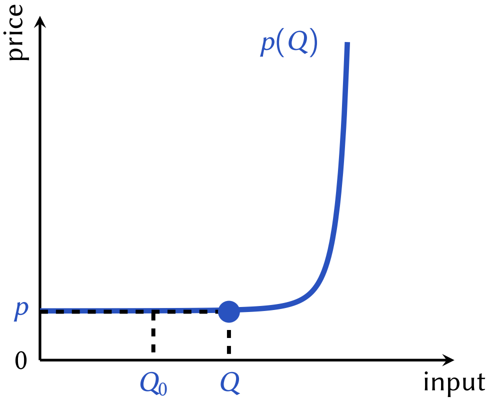

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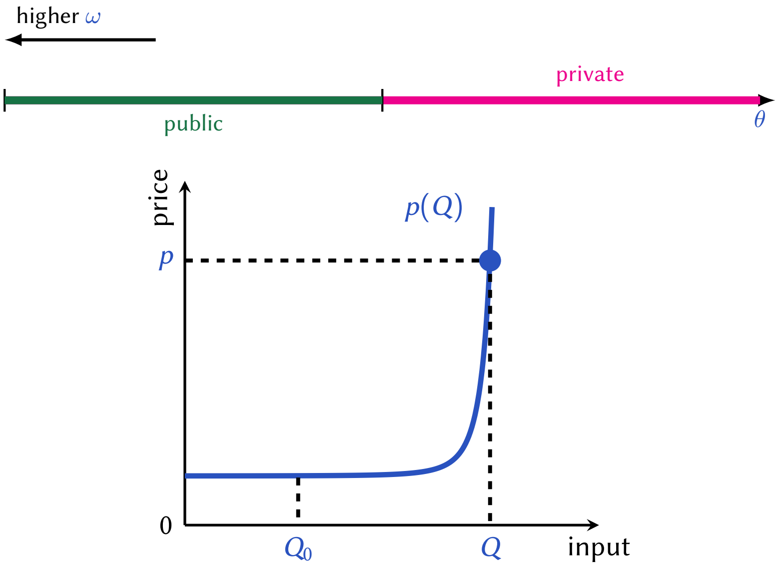

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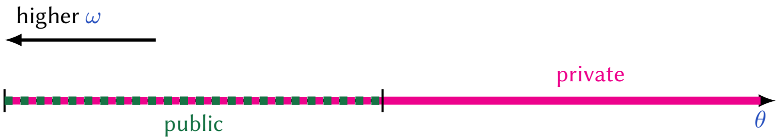

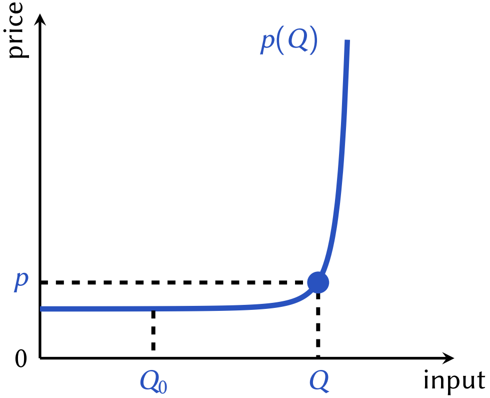

Discussion

Many assumptions can be relaxed:

- **Unit demand/supply.**

  Agents need not have unit demand/supply (*e.g.*, Kang and Muir, 2022).

Extensions

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  Objective and constraint(s) need not be affine in x (*e.g.*, Kang, 2022).

- **Market structure assumptions.**

  Aftermarket need not be perfectly competitive (*e.g.*, Kang, 2023).

Takeaway #1: equilibrium effects are important in many settings

Trade-offs in other economic problems can be understood via direct and indirect effects.

Problem	Instruments	Indirect feedback
contracting with a downstream market	allocation of input	price of final good
redistribution with a private market	allocation of public option	price of input good

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Other problems: adverse selection, imperfect competition, costly search, market frictions…

Other instruments: taxes/subsidies on multiple goods, price controls, product specification regulation…

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- "Rationing arises only for unusual distributions; 'standard' problems don't require it."

Now: equilibrium effects can necessitate optimal rationing.

Conclusion

This tutorial: an approach for mechanism design problems with equilibrium effects.

- This approach involves (only slightly) modifying existing mechanism design tools.
- Equilibrium effects are important; can lead to new insights on optimal mechanisms.
- Many problems untouched; many potentially exciting and new areas for research!

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