Inequality-Aware Partial Market Design

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Motivation

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IMD: How to allocate scarce resources in the presence of inequality and incomplete info?

IPMD: What changes if the social planner can design only part of the market?

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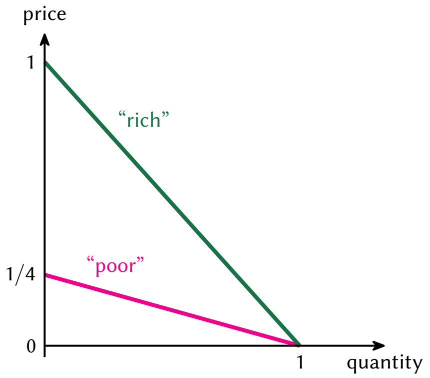
$$D_H(p) = 1 - p \quad \text{and} \quad D_L(p) = 1 - 4p.$$



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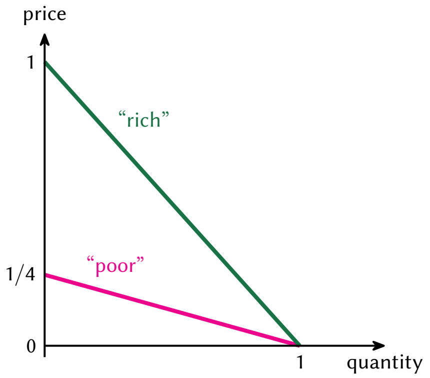

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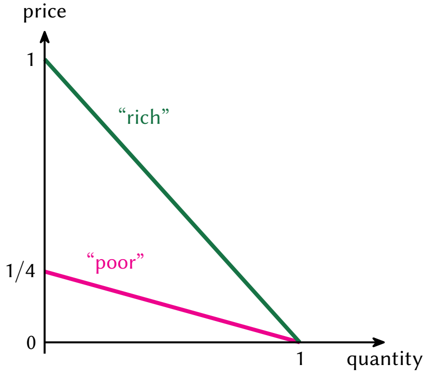

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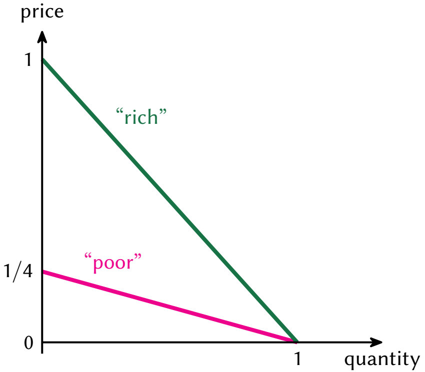
- The social planner assigns weights $\mathbf{E}[\omega] = 1$ to producers and public expenditure.



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- Under full design, the social planner taxes consumption to redistribute to the poor.
- If the poor had access to the private market, they would purchase positive quantities!

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- Equilibrium effects on price complicate the design of optimal redistribution.

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Intuition:

- Topping up changes the outside options of participants of the redistribution program.

Two Challenges

As the example shows, there are two main challenges in partial market design:

1. **Equilibrium effects**

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1. **Equilibrium effects**

   Without equilibrium effects, the price is exogenously given.

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2. **Outside options**

   Design choices (e.g., topping up) changes consumers' outside options.

   Consumers' outside options are type-dependent, presenting a technical challenge.

Equilibrium Effects

based on “The Public Option and Optimal Redistribution”

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In recent years, **public options** have gained traction in policy discussions.

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Question. How do equilibrium effects shape optimal redistribution with a public option?

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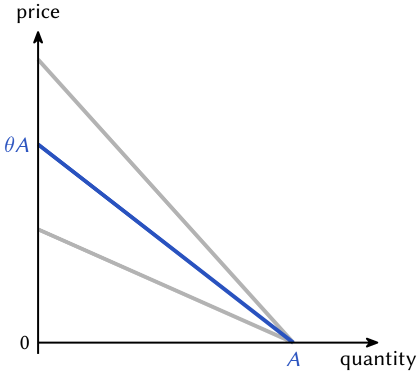
  $\Theta \subset \mathbb{R}_+$ is compact, and types θ have a CDF F, which has full support on Θ.
- Each consumer derives utility $u(q, \theta) - t$ from quantity $q \in [0, A]$ at total price t.

  Assume that $u(\cdot, \theta) : [0, A] \to \mathbb{R}$ is increasing, strictly concave, and satisfies strict SCP.

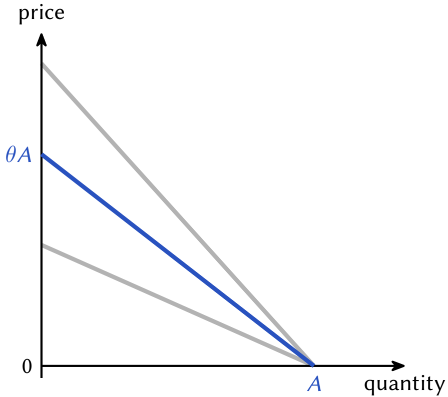
  Example: $u(q, \theta) = \theta \left(Aq - \frac{1}{2}q^2\right)$.



- Each consumer solves

$$D(p,\theta) \in \arg\max_{q\in[0,A]} \left[u(q,\theta) - pq\right].$$

Since u is strictly concave in q, there is a unique maximizer.



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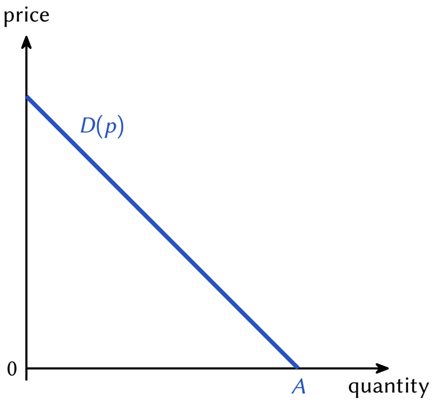

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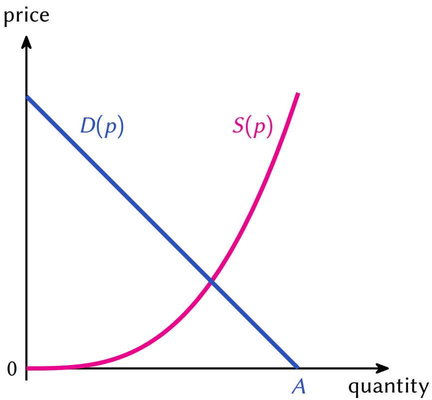

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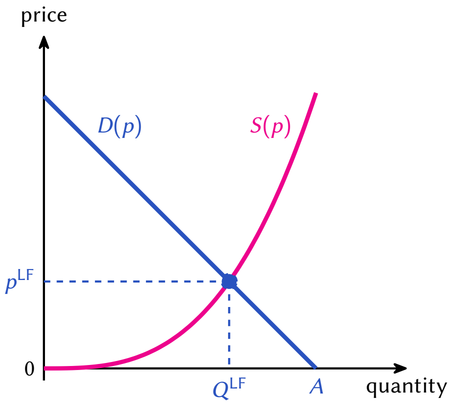

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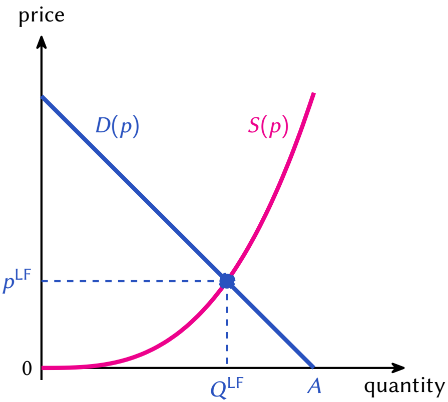
- Expanding aggregate demand increases price!



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- Formally, the social planner chooses a Borel measure $\mu \in \Delta(E)$, where $E = [0, A]$:
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- μ is **deterministic** if $\mu = \delta_e$ for some $e \in E$ (i.e., the lottery is degenerate).

  Otherwise, μ is **randomized** (i.e., the lottery is nondegenerate).

Timing

#1. The social planner chooses an allocation rule $\mu \in \Delta(E)$.

#2. Consumers (voluntarily) apply for the public option, and entitlements are realized.

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The equilibrium price p satisfies

$$S(p) = \int_E \int_\Theta \max\{e, D(p,\theta)\} \, \mathrm{d}F(\theta) \, \mathrm{d}\mu(e).$$

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- **Public expenditure**: a dollar of public expenditure has social cost $\alpha \in \mathbb{R}_+$,

$$C(\mu; Q) = \alpha P(Q) \int_E e \,\mathrm{d}\mu(e).$$

Related Work

- **Redistributive Mechanism Design.** Che, Gale and Kim (2013); Condorelli (2013); Dworczak ⓡ Kominers ⓡ Akbarpour (2021); Akbarpour ⓡ Dworczak ⓡ Kominers (2022); Akbarpour ⓡ Dworczak ⓡ Kominers (2024); Pai and Strack (2024); Kang and Watt (2026); ...
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- **Methodological Tools.**
  - Concavifcation: Aumann and Maschler (1995); Kamenica and Gentzkow (2011); ...
  - ⤳ **This paper:** market clearing is mathematically equivalent to a "Bayes plausibility" constraint.

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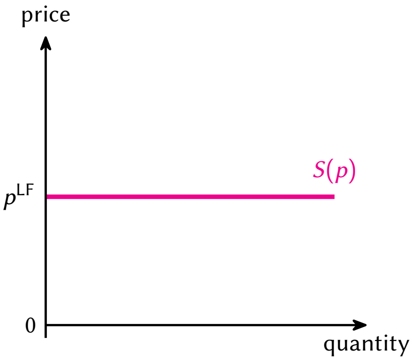

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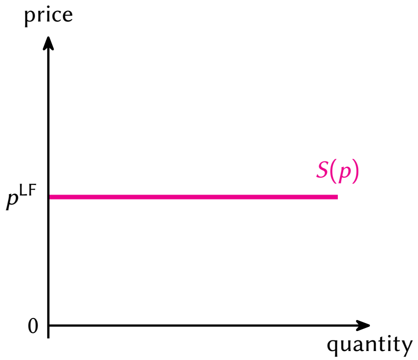
- When supply is perfectly elastic, there are no equilibrium effects or randomization.
- Thus, equilibrium effects can lead to optimal randomization (and justify randomization).



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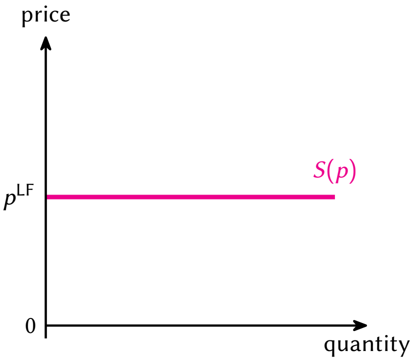
- When supply is perfectly elastic, there are no equilibrium effects or randomization.
- Thus, equilibrium effects can lead to optimal randomization (and justify randomization).
- This challenges proposals that advocate for "**universality**" (Sitaraman and Alstott, 2019).
  - $\rightsquigarrow$ Nonuniform entitlements may be better!



Main Result #2: Zero Entitlements Can Be Optimal

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- This challenges proposals that advocate for using the public option as a "**baseline**."

Analysis: Direct Effect vs. Indirect Effect

Write weighted total surplus (i.e., the social planner's objective) as

$$\mathrm{TS}(\mu; Q) = \mathrm{CS}(\mu; Q) + \mathrm{PS}(Q) - C(\mu; Q).$$

In equilibrium, the aggregate quantity Q satisfies

$$Q = \int_E \int_\Theta \max\{e, D(P(Q), \theta)\} \, \mathrm{d}F(\theta) \, \mathrm{d}\mu(e).$$

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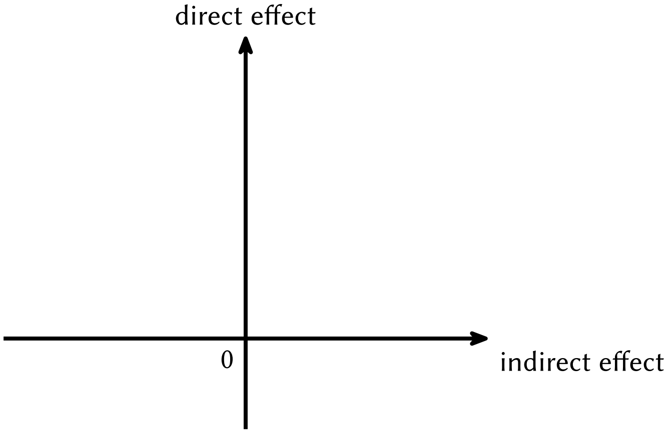
The change in weighted total surplus from laissez-faire is

$$\begin{aligned}
&\mathsf{TS}(\mu; Q) - \mathsf{TS}(\delta_0; Q^{\mathsf{LF}}) \\
&= \underbrace{\mathsf{TS}(\mu; Q) - \mathsf{TS}(\delta_0; Q)}_{\textbf{direct effect} \text{ (holding fixed aggregate quantity at } Q\text{)}} + \underbrace{\mathsf{TS}(\delta_0; Q) - \mathsf{TWS}(\delta_0; Q^{\mathsf{LF}})}_{\textbf{indirect effect} \text{ (due to change in aggregate quantity)}} .
\end{aligned}$$

Key Trade-Off: Direct Effect vs. Indirect Effect ▸ more



TS = **direct effect** (aggregate quantity $= Q$) + **indirect effect** (aggregate quantity $Q^{\mathsf{LF}} \rightarrow Q$).

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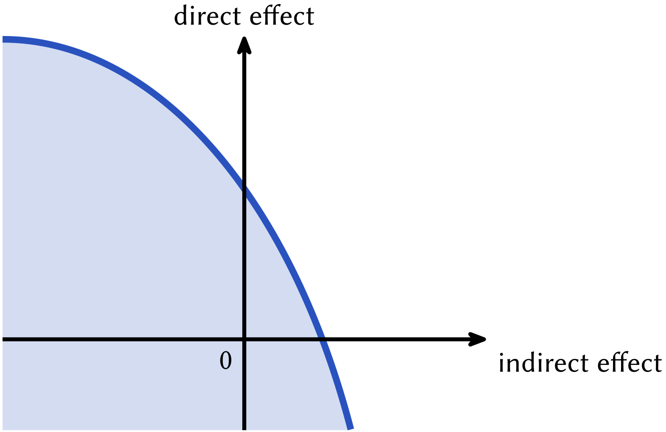

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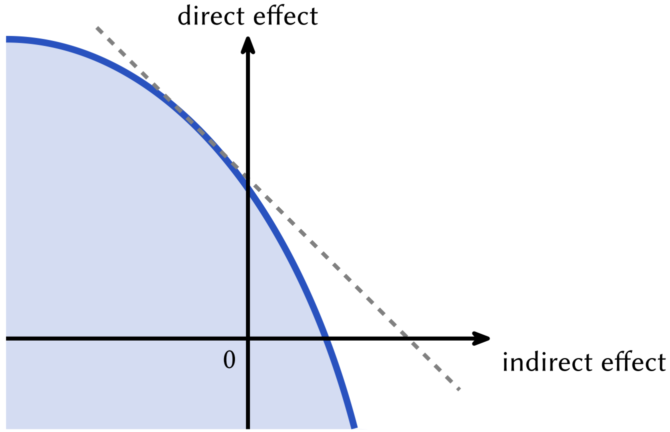

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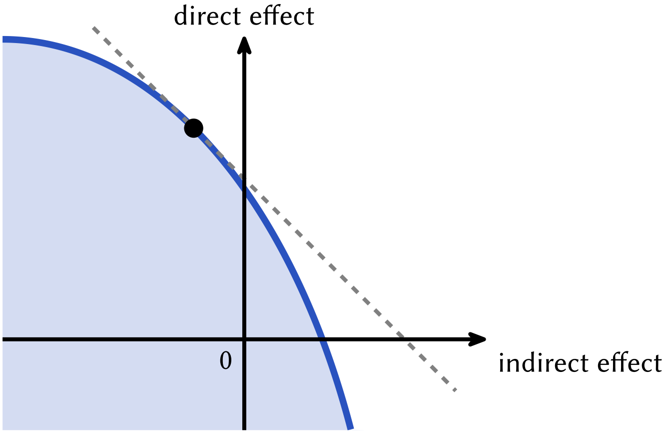

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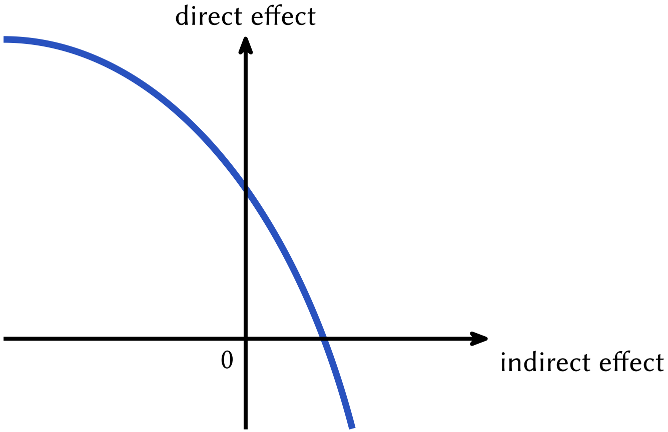
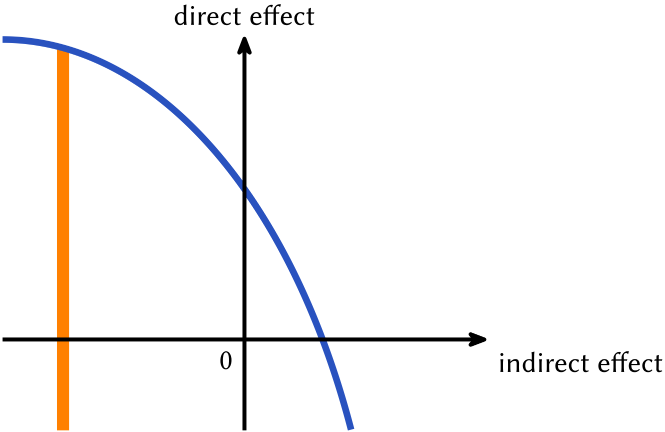
Solving the Social Planner's Problem ▸ more



Choose allocation that maximizes direct effect at any given value of the indirect effect.

Solving the Social Planner's Problem ▸ more



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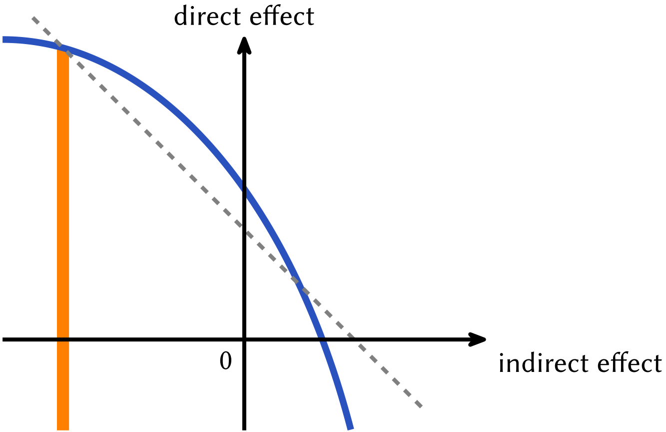

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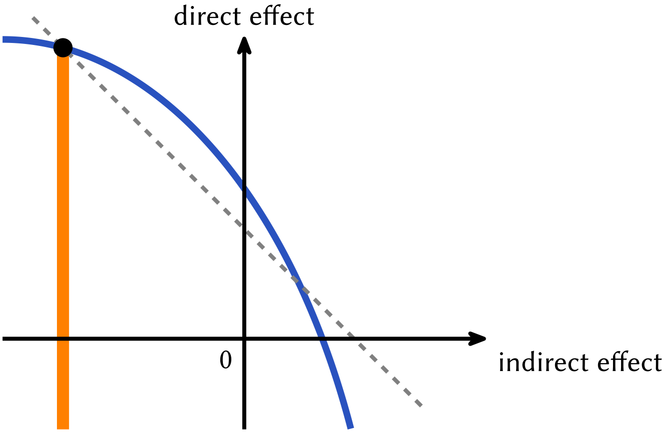

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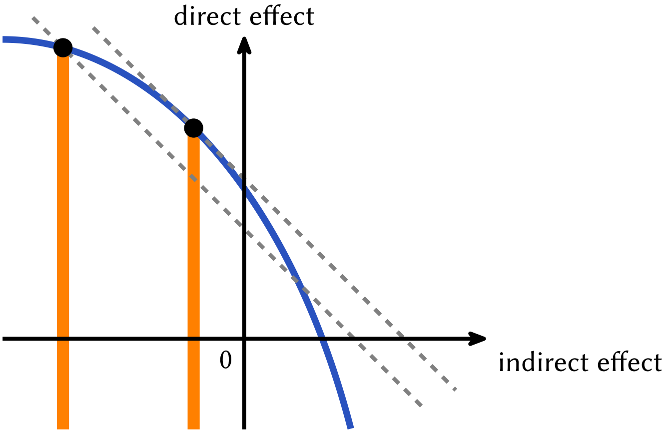
Solving the Social Planner's Problem ▸ more



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$$\mathsf{CS}(\mu; Q) - C(\mu; Q) = \int_E \int_\Theta [V(e, \theta; Q) - \alpha e P(Q)] \, \mathrm{d}F(\theta) \, \mathrm{d}\mu(e).$$

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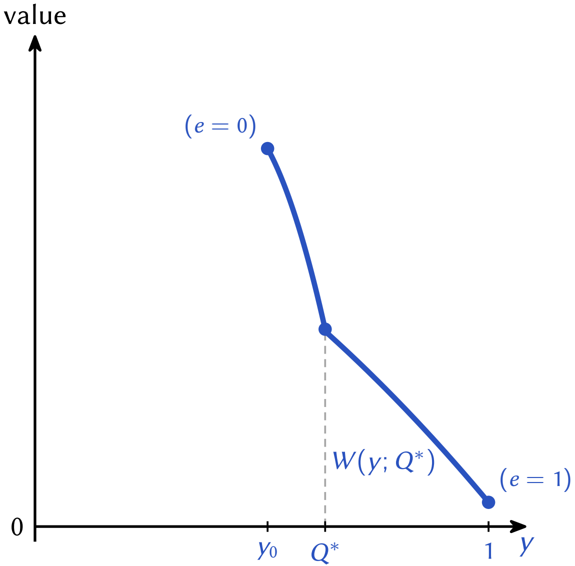
The objective is

$$W(y;Q) := \int_\Theta \left[V(e(y),\theta;Q) - \alpha e(y)P(Q)\right] \mathrm{d}F(\theta).$$

This is **consumer surplus − public expenditure**, in units of **total consumption** (y).

Solving the Social Planner's Problem



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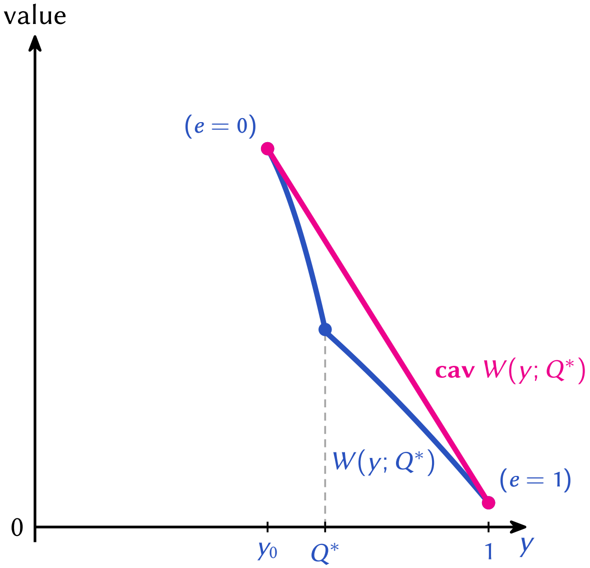

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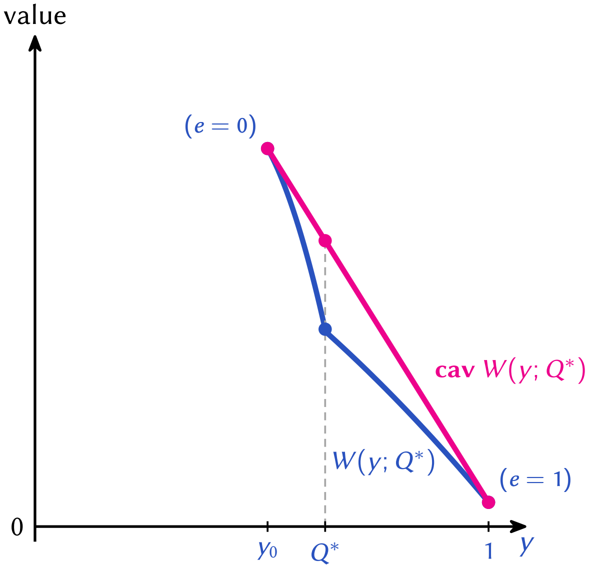
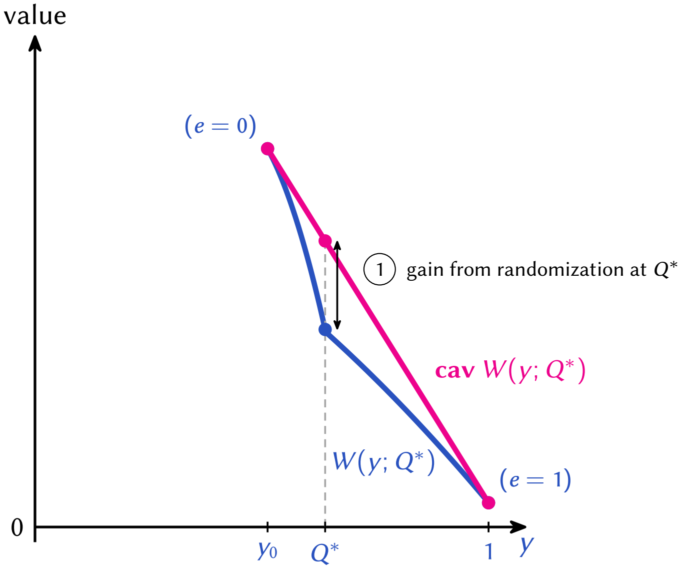
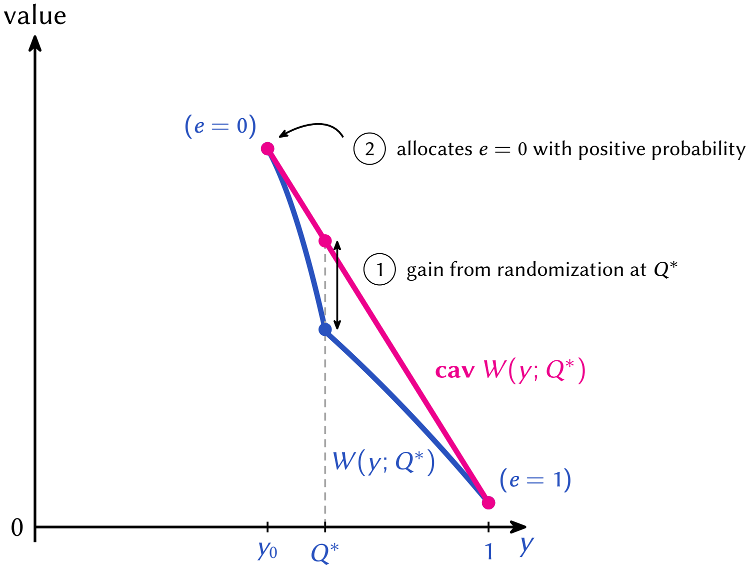



value
$(e = 0)$
① gain from randomization at Q^*
cav $W(y; Q^*)$
$W(y; Q^*)$
$(e = 1)$
0
y_0
Q^*
1
y

Solving the Social Planner's Problem



Theorem. A **randomized** allocation is strictly optimal iff $W(Q^*; Q^*) \neq \textbf{cav}\, W(Q^*; Q^*)$ for every

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Main Results Revisited

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Theorem. Define

$$\textbf{rad}\, W(z; Q) := z \sup_{y \geq z} \frac{W(y; Q)}{y}.$$

Every optimal allocation rule **randomizes over positive entitlements** iff $\textbf{rad}\, W(Q^*; Q^*) < \textbf{cav}\, W(Q^*; Q^*)$ for every

$$Q^* \in \arg\max_Q \left[\mathrm{PS}(Q) + \textbf{cav}\, W(Q; Q)\right].$$

Summary

① Equilibrium effects can lead to **optimal randomization**.

Without equilibrium effects, the optimal allocation rule is deterministic.

② Equilibrium effects can lead to **zero entitlements** conditional on intervention.

Without equilibrium effects, entitlements are positive conditional on intervention.

In the paper, sufficient conditions are derived in terms of **elasticity of supply**.

Outside Options

based on “Topping Up and Optimal Redistribution” (with Mitchell Watt)

Motivation

Governments often redistribute through in-kind programs:

- food stamps (e.g., SNAP); and
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Topping up is often a design choice of in-kind programs and affects screening:

- Unlike public housing units, housing vouchers would allow topping up (if desired).
- While recipients benefit, topping up weakens screening by expanding participation.

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> —Currie and Gahvari (2008)

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Question. How does topping up affect the optimal design of redistribution programs?

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  The good is produced competitively at a constant marginal cost, $c > 0$.

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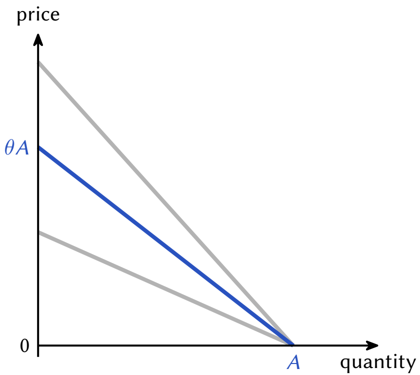
  Types θ have a CDF F, which is continuously differentiable with density $f > 0$ on $[\underline{\theta}, \overline{\theta}]$.
- Each consumer derives utility $\theta v(q) - t$ from quantity $q \in [0, A]$ at total price t.

  Assume that $v : [0, A] \to \mathbb{R}$ is increasing and strictly concave (with $v' > 0$ and $v'' < 0$).

  Example: $v(q) = Aq - \frac{1}{2}q^2$ (linear demand).

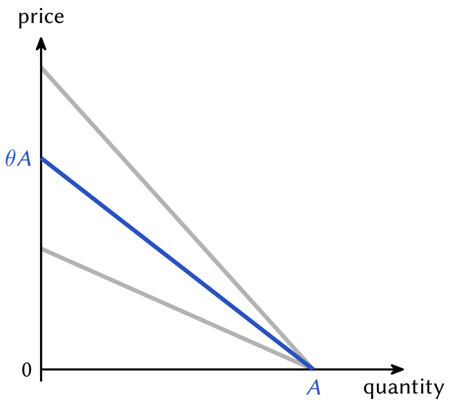


Laissez-Faire Equilibrium

- Since the private market is perfectly competitive, price $=$ marginal cost $= c$.
- Each consumer solves

$$U^{\mathsf{LF}}(\theta) := \max_{q \in [0,A]} \left[\theta v(q) - cq\right].$$

Since v is strictly concave, there is a unique maximizer $q^{\mathsf{LF}}(\theta)$, assumed to be interior.



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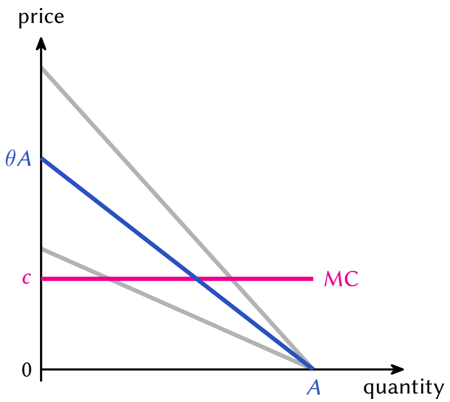

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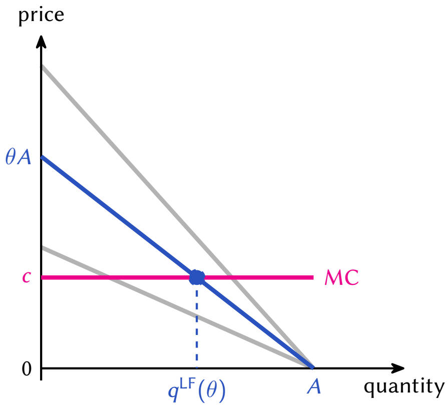

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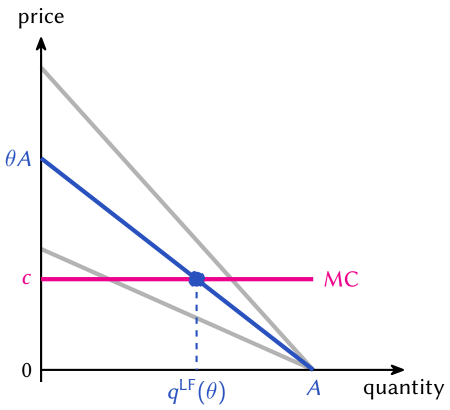
- Assumption of constant MC can be relaxed.
  - Upward-sloping supply: Kang (2023).
    Eqm. effects $\Rightarrow$ alt. redistribution channel.
  - **This paper:** participation constraints.



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  - the allocation function $q : [\underline{\theta}, \overline{\theta}] \to [0, A]$ denoting *total* consumption by consumer;
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- We focus on **in-kind** programs: $t(\cdot) \geq 0 \quad \leadsto$ no lump-sum transfers to consumers.

  However, we allow for lump-sum transfers to consumers "outside of the mechanism."

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The social planner maximizes **weighted total surplus**, which consists of:

- **Consumer surplus**

- **Total profit**

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  - $\rightsquigarrow$ $\omega(\theta)$: expected social value of giving him one unit of money.
  - $\rightsquigarrow$ Monotonicity of ω captures the correlation between demand and redistributive priority.

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For any given mechanism (q, t), the weighted total surplus can be written as:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} + \alpha \underbrace{[t(\theta) - cq(\theta)]}_{\text{total profit}} \right] \mathrm{d}F(\theta).$$

Mechanism Design

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subject to

- incentive compatibility, $$\theta \in \arg\max_{\hat{\theta} \in [\underline{\theta}, \overline{\theta}]} \left[\theta v(q(\hat{\theta})) - t(\hat{\theta}) \right], \quad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]; \quad \text{(IC)}$$
- no lump-sum transfers, $$t(\theta) \geq 0, \quad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]; \quad \text{(LS)}$$
- individual rationality, $$\theta v(q(\theta)) - t(\theta) \geq U^{\mathrm{LF}}(\theta), \quad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]; \quad \text{(IR)}$$
- topping up, $$q(\theta) \geq q^{\mathrm{LF}}(\theta), \quad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]. \quad \text{(TU)}$$

Related Work

- **Redistributive Mechanism Design.** Che, Gale and Kim (2013); Condorelli (2013); Dworczak ⓡ Kominers ⓡ Akbarpour (2021); Akbarpour ⓡ Dworczak ⓡ Kominers (2022); Akbarpour ⓡ Dworczak ⓡ Kominers (2024); Pai and Strack (2024).
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- **Methodological Tools in Mechanism Design.**
  - Generalized ironing: Toikka (2011).
  - Lagrangian approach: Amador, Werning and Angeletos (2006); Amador and Bagwell (2013).
  - Type-dependent outside options: Jullien (2000), Dworczak and Muir (2024).
  - Majorization constraints (in a concave program): Kleiner, Moldovanu and Strack (2021).
  - $\rightsquigarrow$ **This paper:** majorization constraints due to type-dependent outside options (in a convex program).

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① When to Optimally Redistribute?

Theorem 1. The optimal mechanism strictly improves on the laissez-faire outcome iff:

$$\begin{cases} \max \omega > \alpha, & \text{if topping up is not allowed,} \\ \max\limits_{\hat{\theta} \in [\underline{\theta}, \overline{\theta}]} \mathbf{E}\left[\omega(\theta) \mid \theta \geq \hat{\theta}\right] > \alpha, & \text{if topping up is allowed.} \end{cases}$$

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- **Negative correlation.** If ω is decreasing, $\max\limits_{\hat\theta \in [\underline{\theta}, \overline{\theta}]} \mathbf{E}\left[\omega(\theta) \mid \theta \geq \hat\theta\right] = \mathbf{E}[\omega] < \max \omega$.

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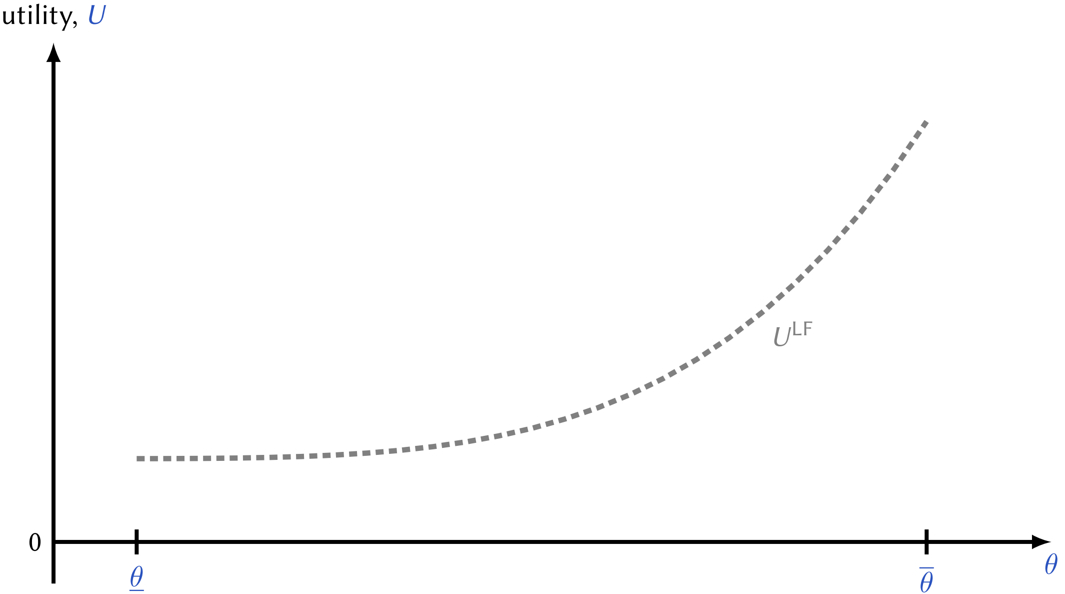

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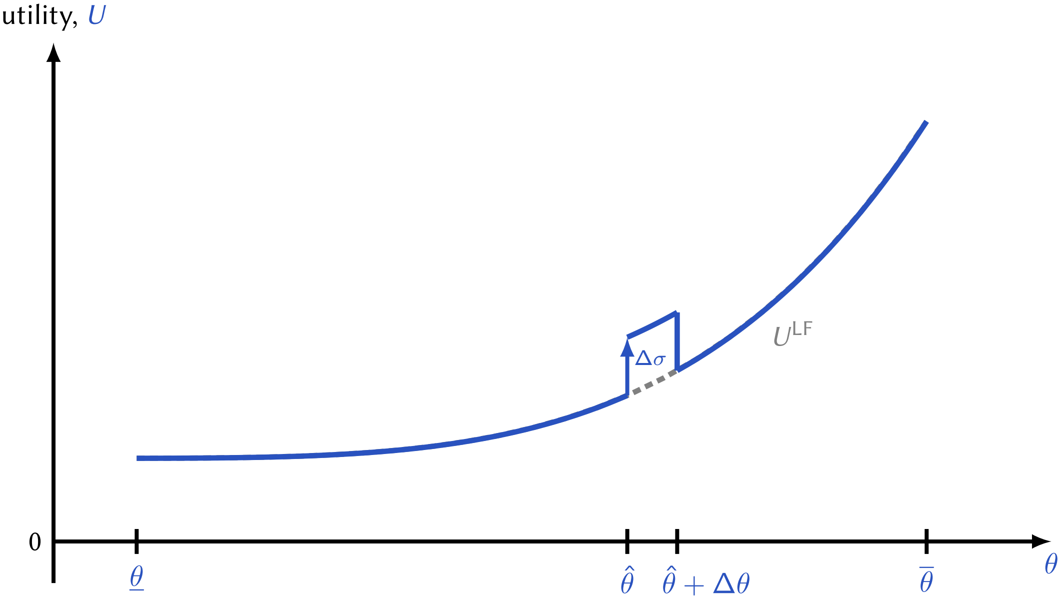

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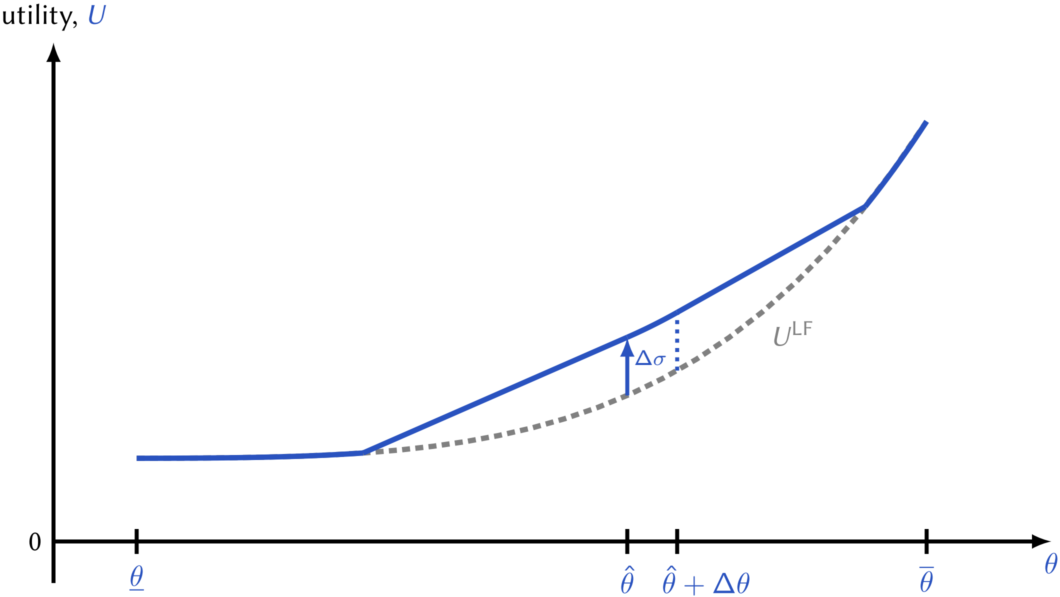

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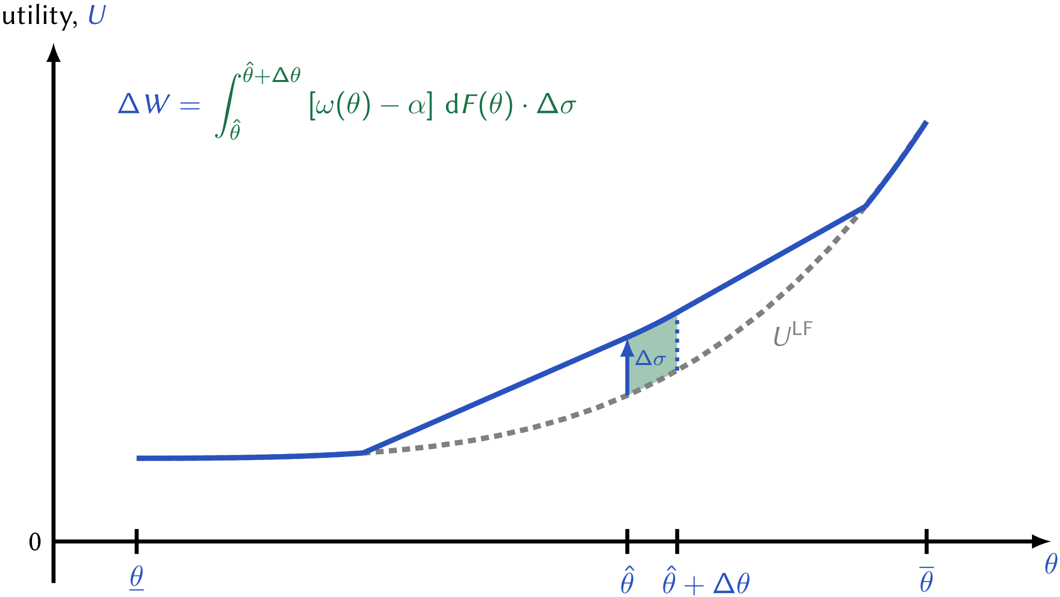

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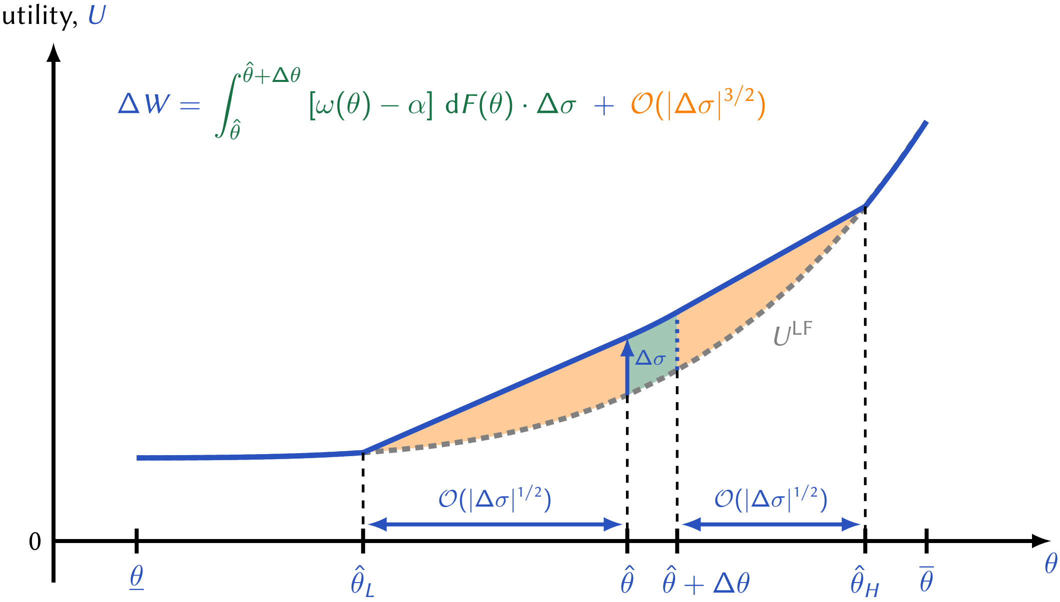

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$$\Delta W = \int_{\hat{\theta}}^{\hat{\theta}+\Delta\theta} [\omega(\theta) - \alpha]\, \mathrm{d}F(\theta) \cdot \Delta\sigma \; + \; \mathcal{O}(|\Delta\sigma|^{3/2})$$

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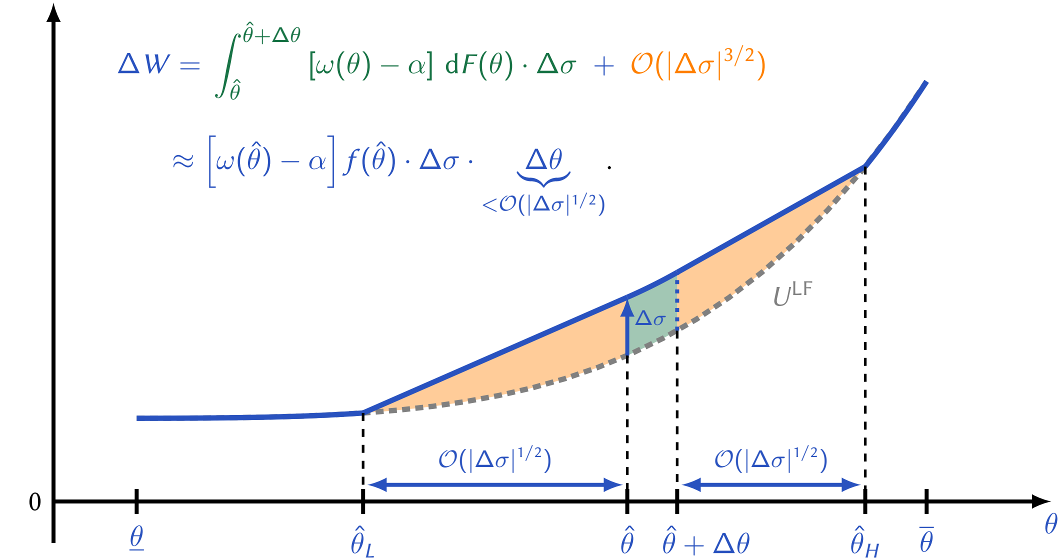

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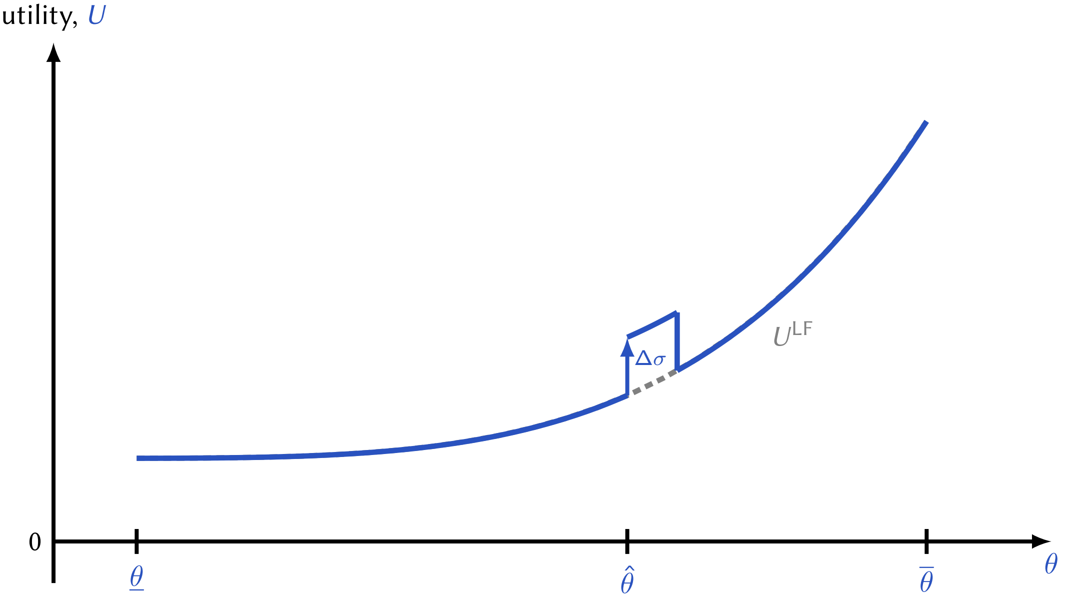

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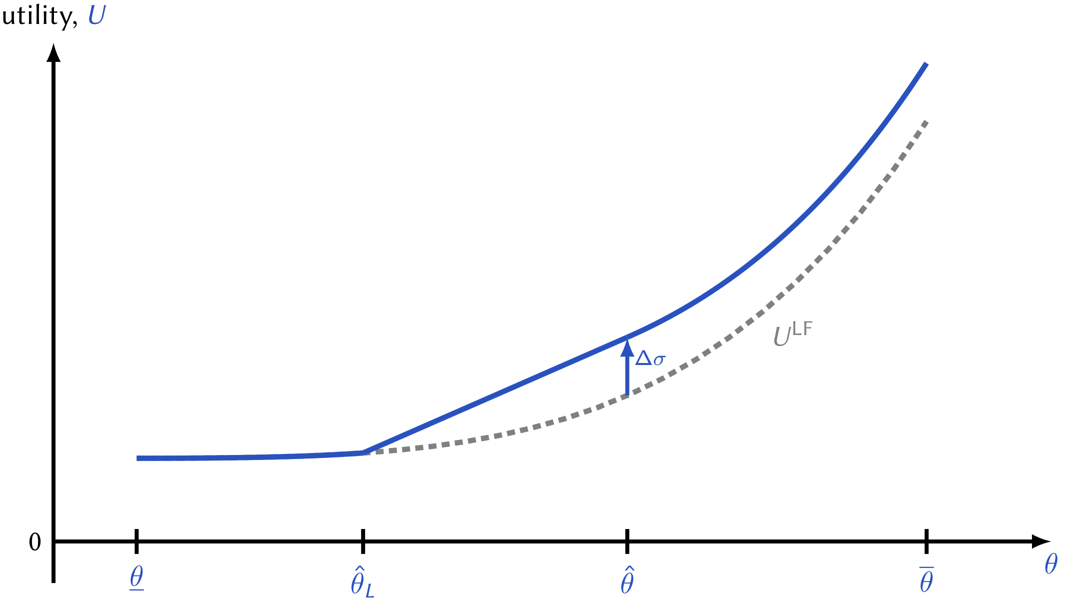

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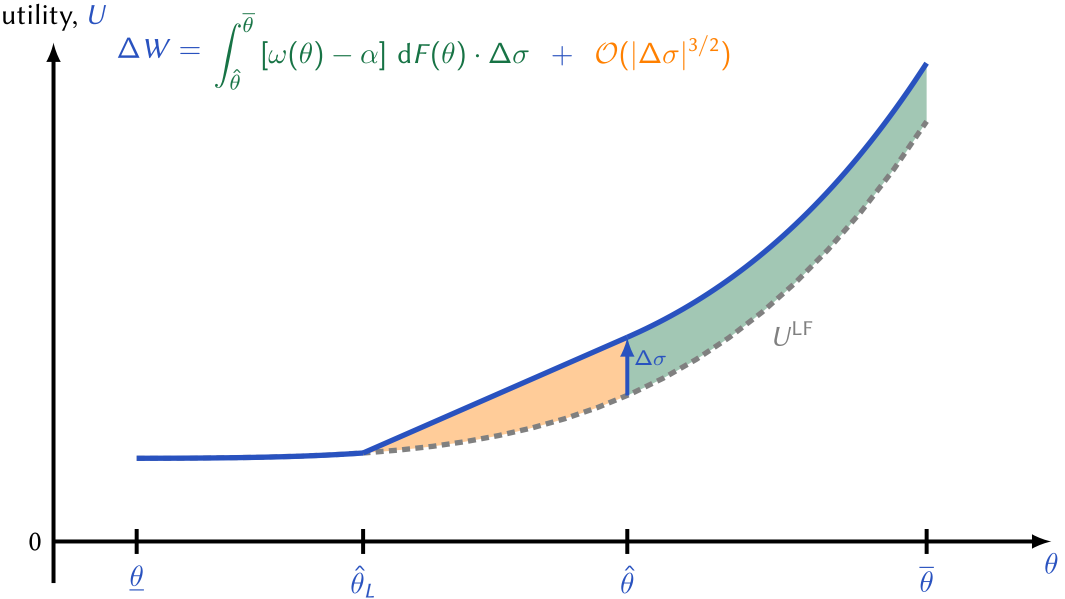

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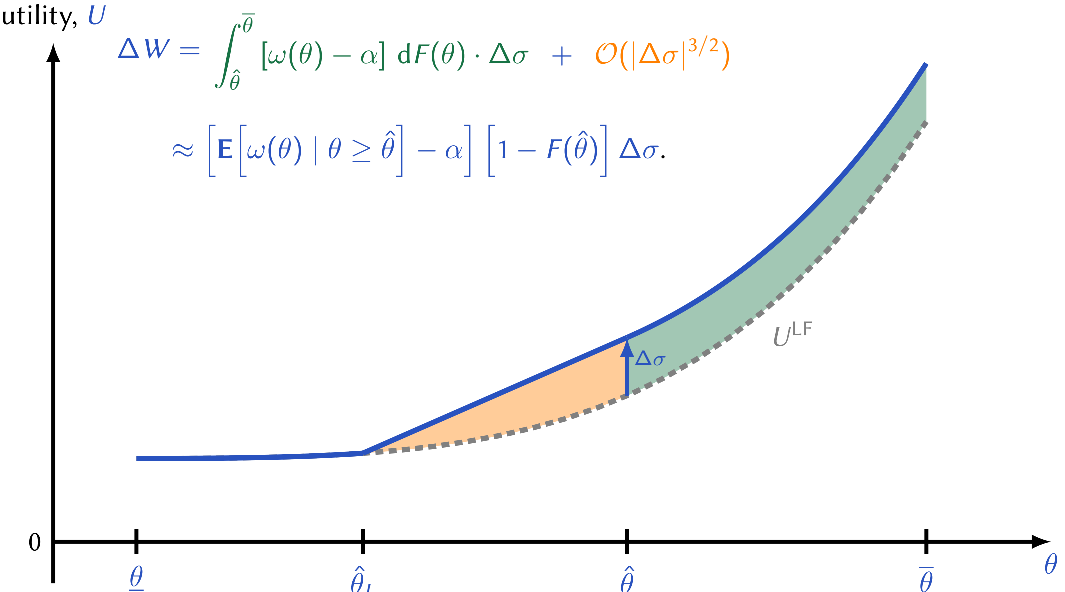

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Corollary. Topping up changes intervention decision iff $\max \omega > \alpha \geq \max_{\hat{\theta}} \mathbf{E}\left[\omega(\theta) \mid \theta \geq \hat{\theta}\right]$.

Theorem 2. Define

$$J_\mu(\theta) := \theta + \frac{\mu\underline{\theta} \cdot \delta_{\underline{\theta}}(\theta) - \int_\theta^{\overline{\theta}} [\alpha - \omega(s)] \,\mathrm{d}F(s)}{\alpha f(\theta)} \quad \text{and} \quad \theta^L := \min\left\{ \hat{\theta} \in [\underline{\theta}, \overline{\theta}] : \int_{\hat{\theta}}^{\overline{\theta}} [\omega(s) - \alpha] \,\mathrm{d}F(s) \geq 0 \right\}.$$

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Topping up can distort screening on the intensive margin, even conditional on intervention.

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Next: how topping up affects **optimal redistribution** for positive vs. negative correlation.

Optimal Mechanisms: Positive Correlation

Theorem 3. When correlation is positive (i.e., ω is increasing), topping up **does not affect** optimal redistribution: the optimal mechanisms with and without topping up coincide.

Intuition:

- Optimal mechanism distorts consumption $\uparrow$ for consumers with high θ ($=$ high ω).
- This is "well-targeted": benefits accrue to consumers with even higher θ ($=$ higher ω).
- If not for topping up, the social planner would provide the "topped-up" units herself.

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Examples: childcare, disability care, public transit, basic staples like coarse bread/cassava.

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The *Public Expenditure Handbook* published by the IMF recommends that:

> "[m]arketed goods with a negative income elasticity (i.e., inferior goods) are ideal candidates for a redistributive subsidy."

Theorem 3 complements the "self-targeting" explanation: no need to prevent topping up.

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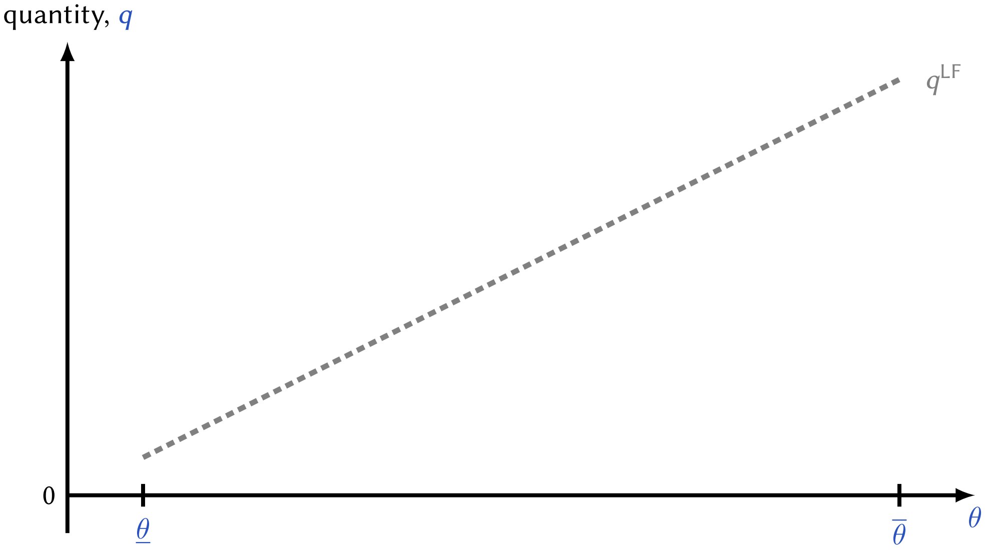

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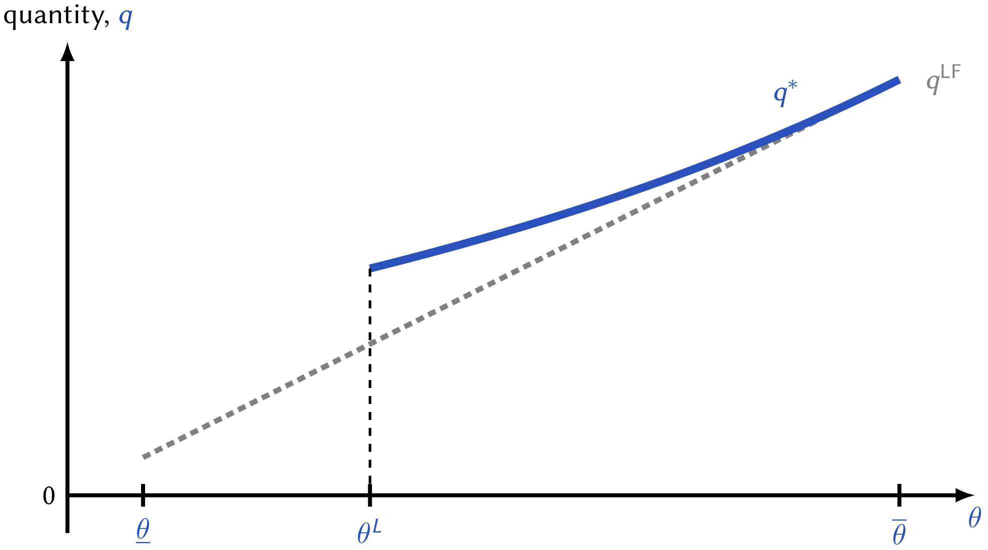

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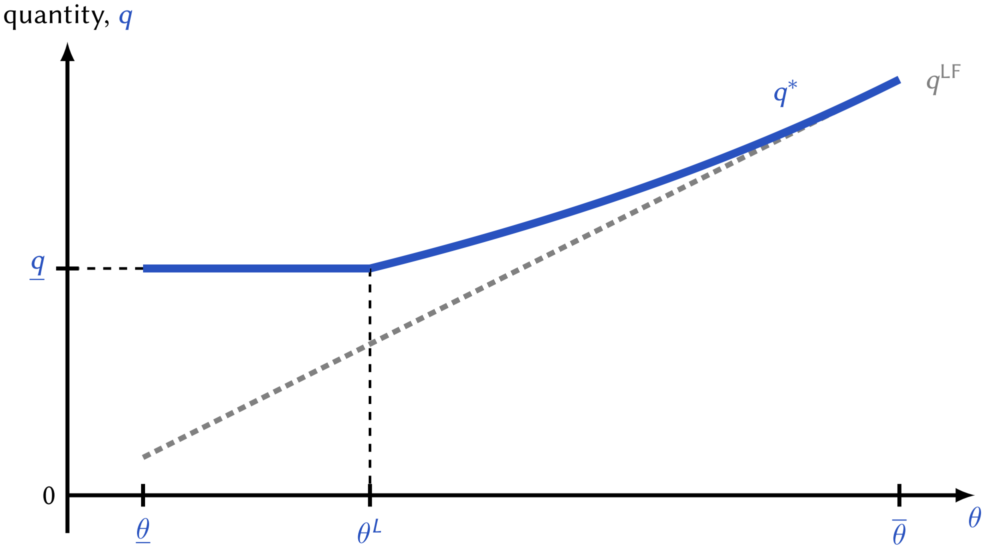

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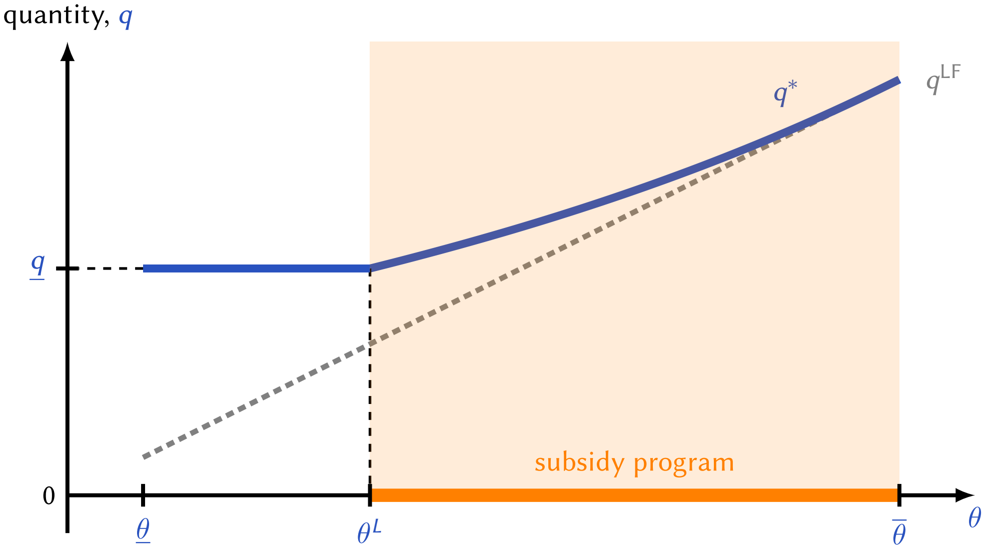

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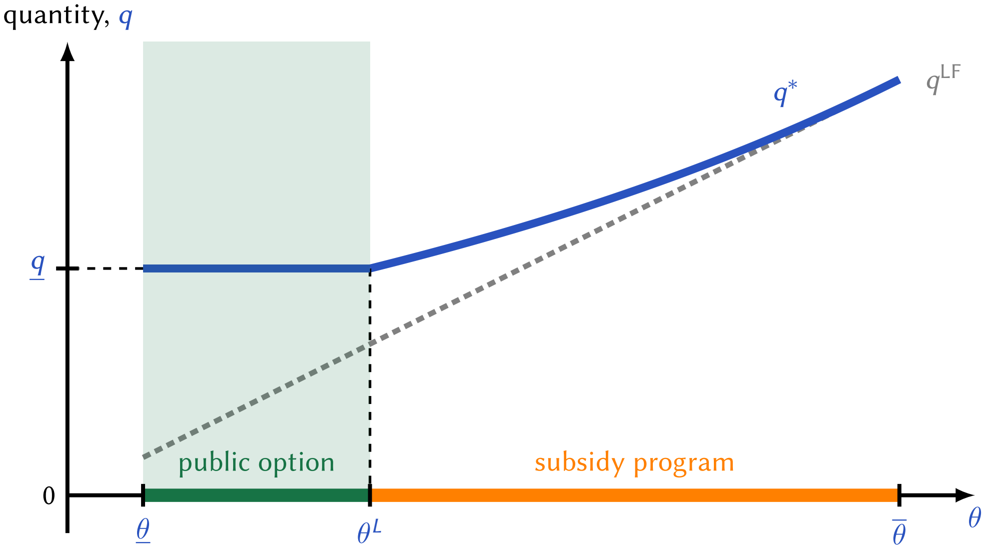

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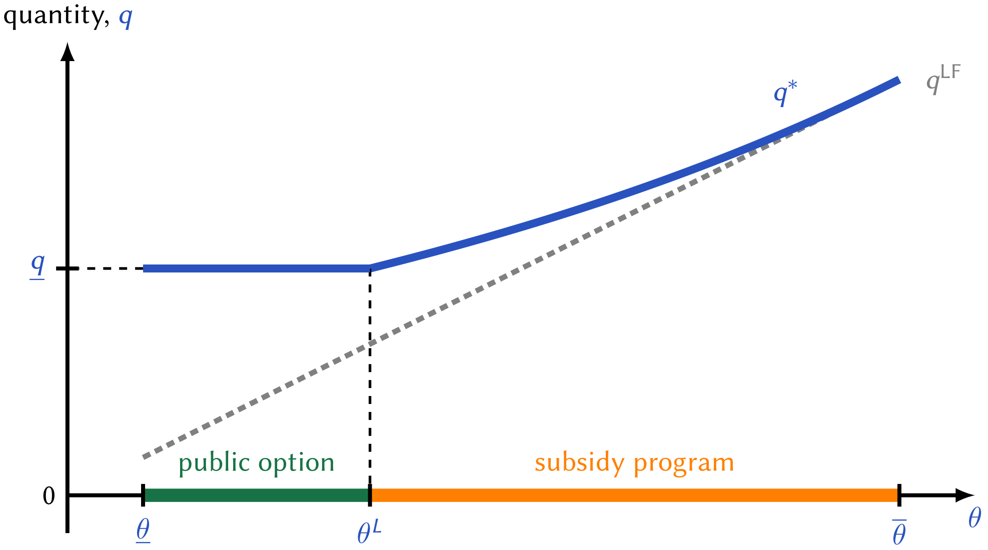

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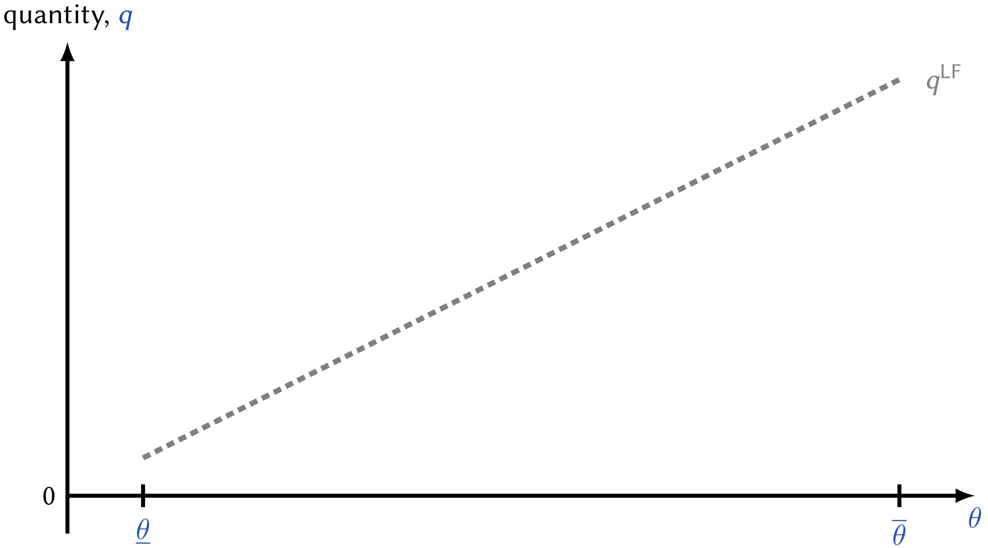

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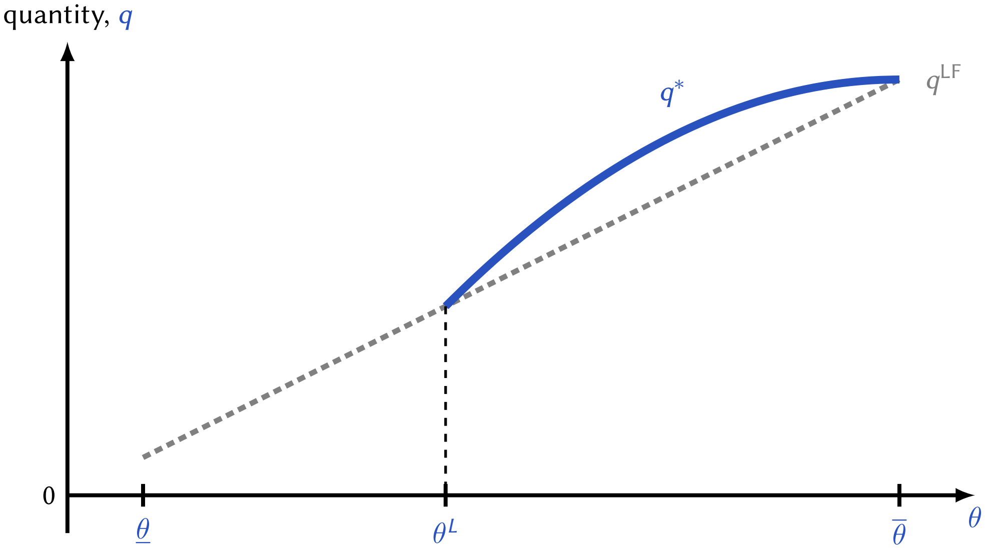

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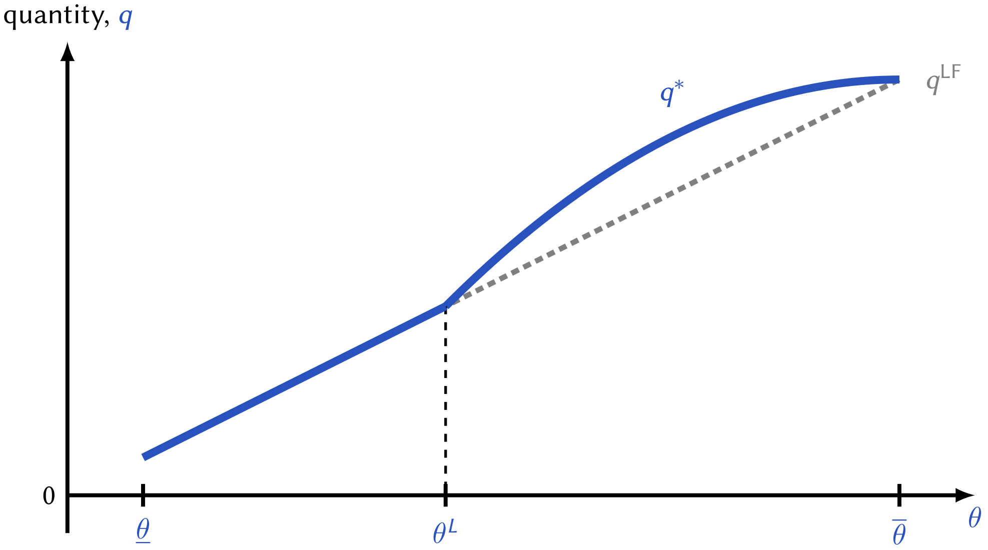

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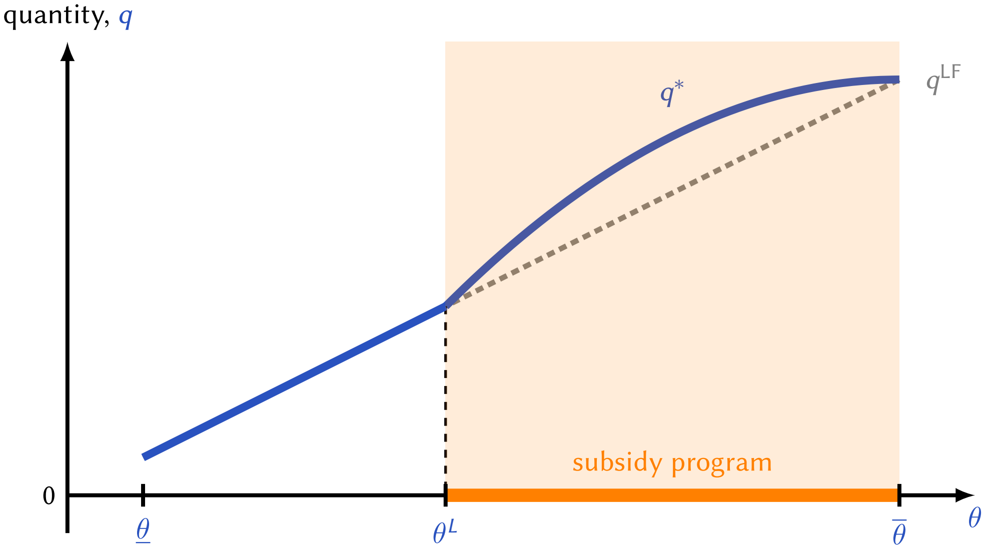

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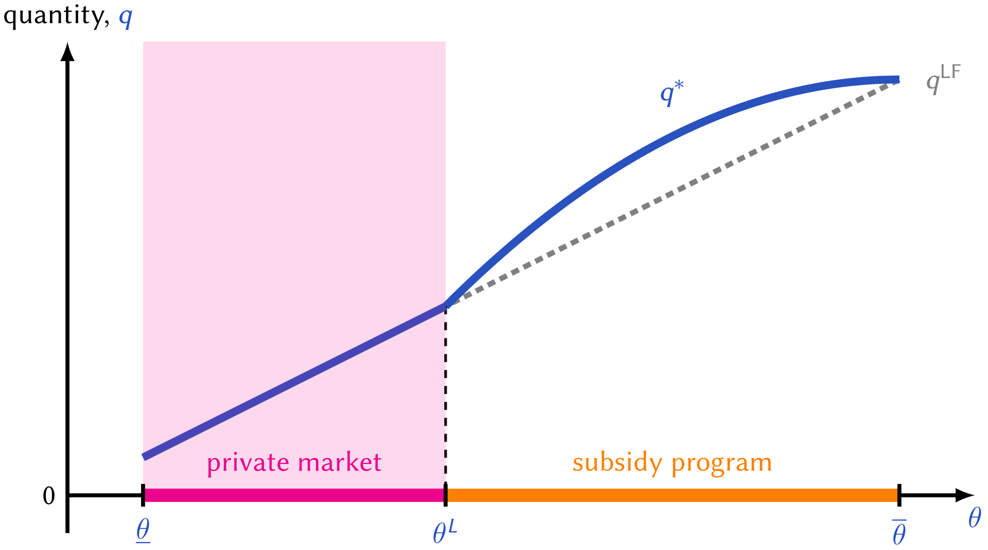

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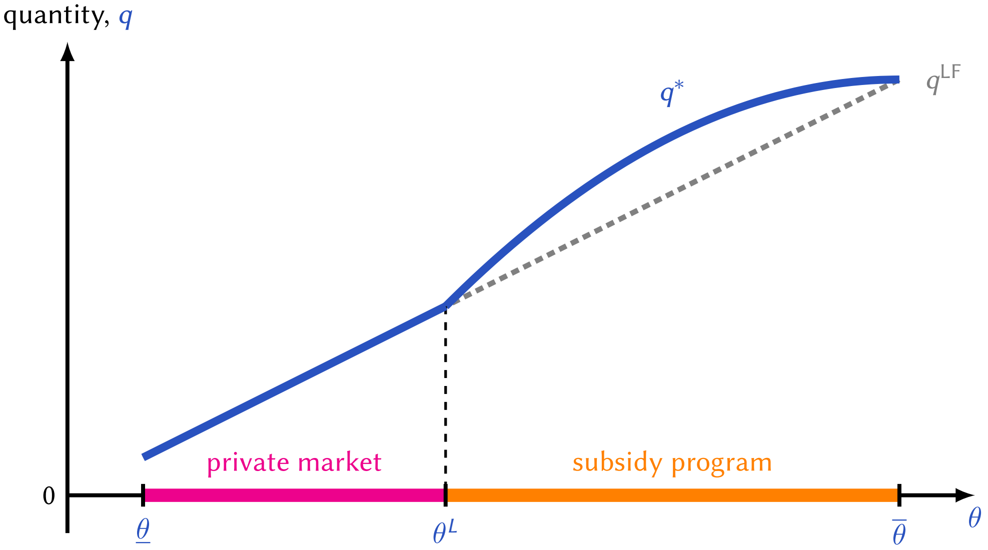

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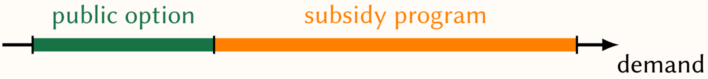

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Optimal Mechanisms: Positive Correlation

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In particular, for any $\mu \geq 0$, define

$$q_\mu(\theta) := D\left(c, \overline{J_\mu}(\theta)\right), \quad \text{where } J_\mu(\theta) := \theta + \frac{\mu\underline{\theta} \cdot \delta_{\underline{\theta}}(\theta) - \int_\theta^{\overline{\theta}} [\alpha - \omega(s)] \, \mathrm{d}F(s)}{\alpha f(\theta)}.$$

Moreover, denote

$$\theta^L := \min\left\{\hat{\theta} \in [\underline{\theta}, \overline{\theta}] : \int_{\hat{\theta}}^{\overline{\theta}} [\omega(s) - \alpha] \, \mathrm{d}F(s) \geq 0\right\}.$$

Let $\mu^* := (\mathbf{E}[\omega] - \alpha)_+$. Then the optimal allocation function is

$$q^*(\theta) = \begin{cases} q_{\mu^*}(\theta) & \text{for } \theta_L \leq \theta \leq \overline{\theta}, \\ q^{\mathrm{LF}}(\theta) & \text{for } \underline{\theta} \leq \theta < \theta^L. \end{cases}$$

Proof Sketch of Theorem 3: Overview

We prove **Theorem 3** in three steps:

#1. Apply basic mechanism design tools $\rightsquigarrow$ constrained mechanism design.

#2. Show that no topping up (IR) constraint binds for lowest type $\underline{\theta}$.

#3. Guess and verify optimal Lagrange multipliers for all other (IR) constraints.

Check that the optimal mechanism also satisfies topping up (TU) constraints.

skip ahead

Step #1: Applying Basic Mechanism Design Tools

The social planner maximizes weighted total surplus:

$$\max_{(q,t)} \int_{\underline{\theta}}^{\overline{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} + \alpha \underbrace{[t(\theta) - cq(\theta)]}_{\text{total profit}} \right] \mathrm{d}F(\theta),$$

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Claim 1 (Myerson, 1981; Milgrom and Segal, 2002). A mechanism (q, t) satisfies (IC) if and only if q is nondecreasing and

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subject to (IR), with the change of variables $\nu(\theta) = v(q(\theta))$.

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Step #2: No Topping Up and Binding (IR) Constraints

The social planner maximizes weighted total surplus:

$$\max_{\substack{\underline{U} \leq \underline{\theta}\nu(\underline{\theta}), \\ \nu \text{ nondecreasing}}} \left\{ [\mathbf{E}[\omega] - \alpha]\, \underline{U} + \int_{\underline{\theta}}^{\overline{\theta}} \left[\left[\alpha\theta - \frac{\int_{\theta}^{\overline{\theta}} [\alpha - \omega(s)]\, \mathrm{d}F(s)}{f(\theta)} \right] \nu(\theta) - \alpha c v^{-1}(\nu(\theta)) \right] \mathrm{d}F(\theta) \right\},$$

subject to

$$\underbrace{\underline{U} + \int_{\underline{\theta}}^{\theta} \nu(s)\, \mathrm{d}s}_{=\theta\nu(\theta)-t(\theta)} \geq \underbrace{\underline{U}^{\mathrm{LF}} + \int_{\underline{\theta}}^{\theta} \nu^{\mathrm{LF}}(s)\, \mathrm{d}s}_{=U^{\mathrm{LF}}(\theta)}, \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]. \tag{IR}$$

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Interpretation: If (IR) binds for some open interval of types (which implies that $\nu=\nu^{\mathsf{LF}}$), then those consumers consume in private market; we say that private market is "active."

Step #2: No Topping Up and Binding (IR) Constraints

Lemma 1. If the private market is active, then the (IR) constraint binds for the lowest type $\underline{\theta}$.

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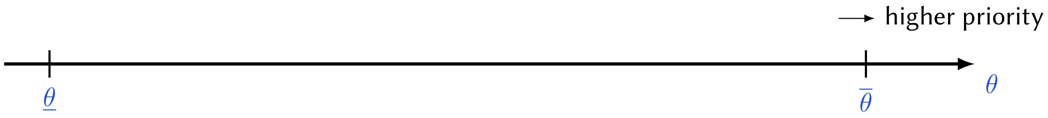

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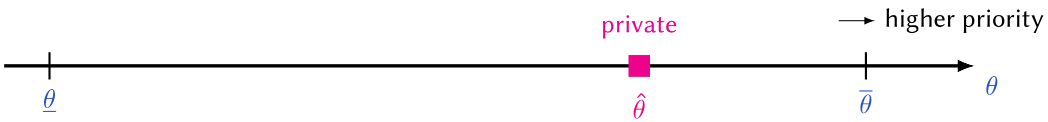

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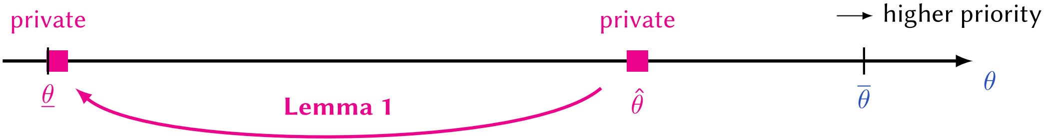

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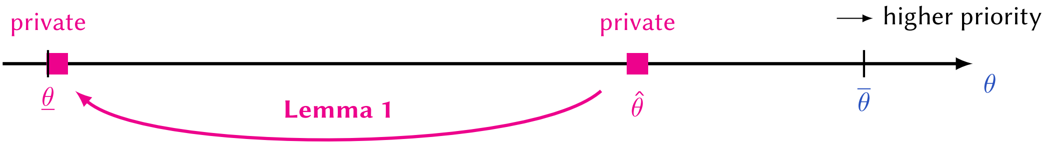

Intuition: The social planner has no incentive to subsidize the lowest priority consumer $\underline{\theta}$ if she already does not subsidize a positive measure of relatively higher priority consumers.

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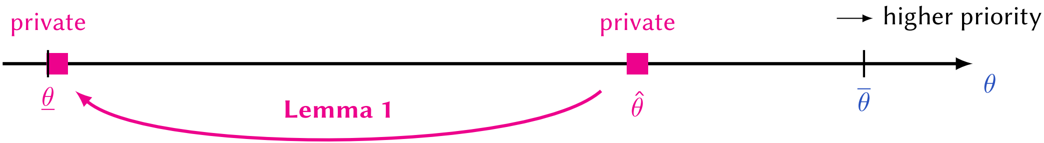

Intuition: The social planner has no incentive to subsidize the lowest priority consumer $\underline{\theta}$ if she already does not subsidize a positive measure of relatively higher priority consumers.

Lemma 2. If the private market is inactive, then the (IR) constraint might bind only for $\underline{\theta}$.

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Given that (IR) binds for lowest type $\underline{\theta}$ (cf. **Lemmas 1** and **2**), $\underline{U} = \underline{U}^{\mathrm{LF}}$.

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If v is affine, then solution is an extreme point (Kleiner et al., 2021; Akbarpour ⓡ al., 2024).

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If v is affine, then solution is an extreme point (Kleiner et al., 2021; Akbarpour ⓡ al., 2024).

Technical difficulty: v is strictly concave, so solution is generally an interior point.

Step #3: Guessing Lagrange Multipliers for (IR) Constraints

To overcome this technical difficulty, we adapt approach of Amador and Bagwell (2013):

#1. Guess a continuum of optimal Lagrange multipliers for (IR) constraints.

#2. Solve for optimal mechanism and verify that (IR) constraints are satisfied.

#3. Verify that complementary slackness conditions are satisfied.

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This allows us to obtain the optimal mechanism with no topping up (**Theorem 3**).

With positive correlation, optimal mechanism also satisfies topping up (IR) constraints.

[With negative correlation, repeat the above approach with topping up (IR) constraints.]

Optimal Mechanisms: Negative Correlation

Theorem 4(a). When correlation is negative (i.e., ω is decreasing), topping up **does not affect** the optimal subsidy program if $\alpha \geq \max \omega$ or $\alpha \leq \min \omega$.

Intuition: If $\alpha \leq \min \omega$, distort consumption $\uparrow$ for all consumers.

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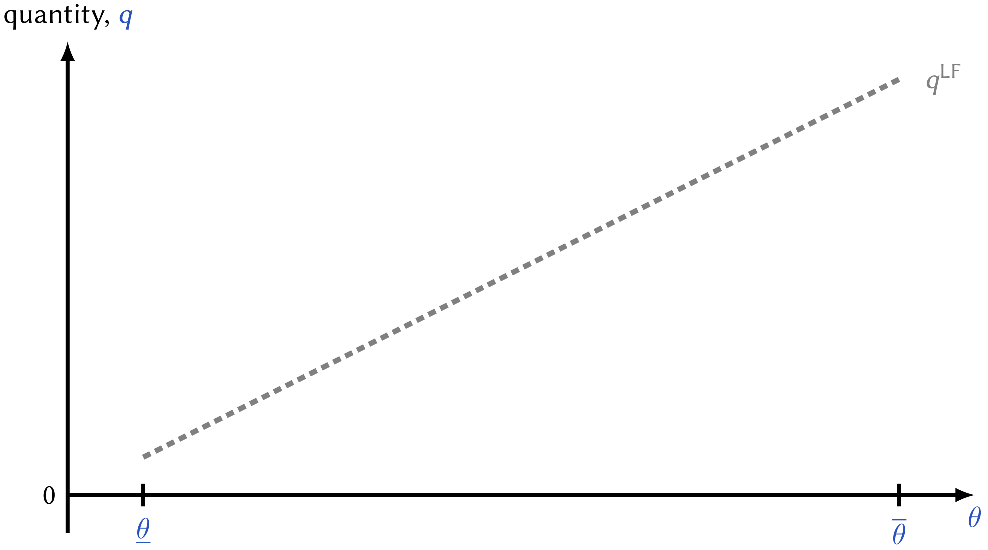
Theorem 4(b). When correlation is negative, there exists $\alpha_{\min} \in [\min \omega, \mathbf{E}[\omega])$ such that topping up **does affect** the optimal subsidy program iff $\alpha_{\min} < \alpha < \max \omega$.

Characterization of Optimal Mechanisms: $\alpha_{\mathsf{min}} < \alpha < \mathbf{E}[\omega]$



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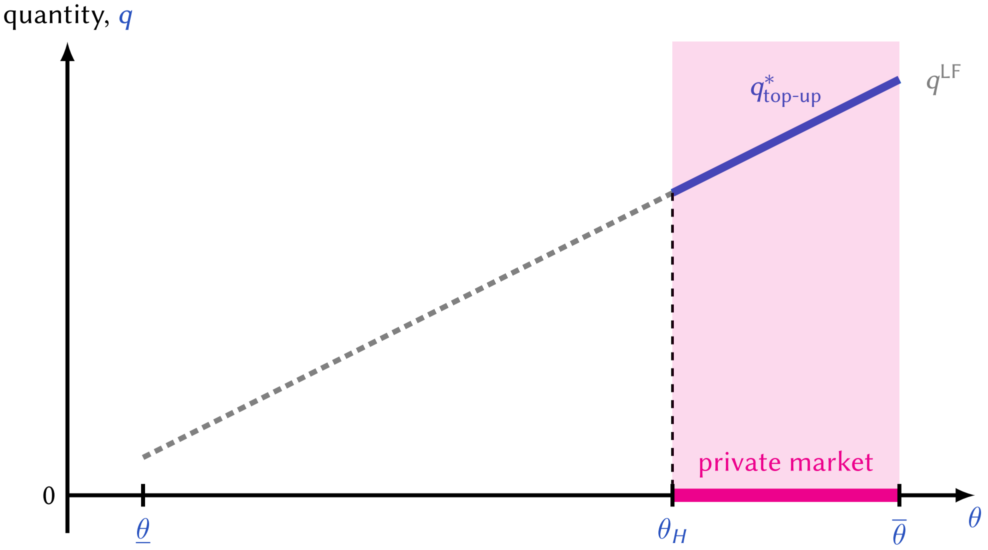

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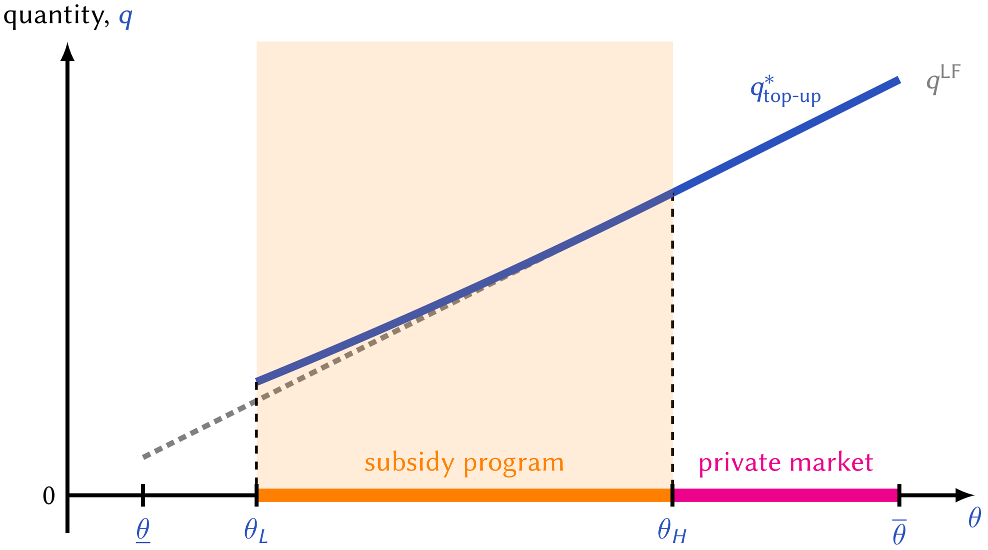

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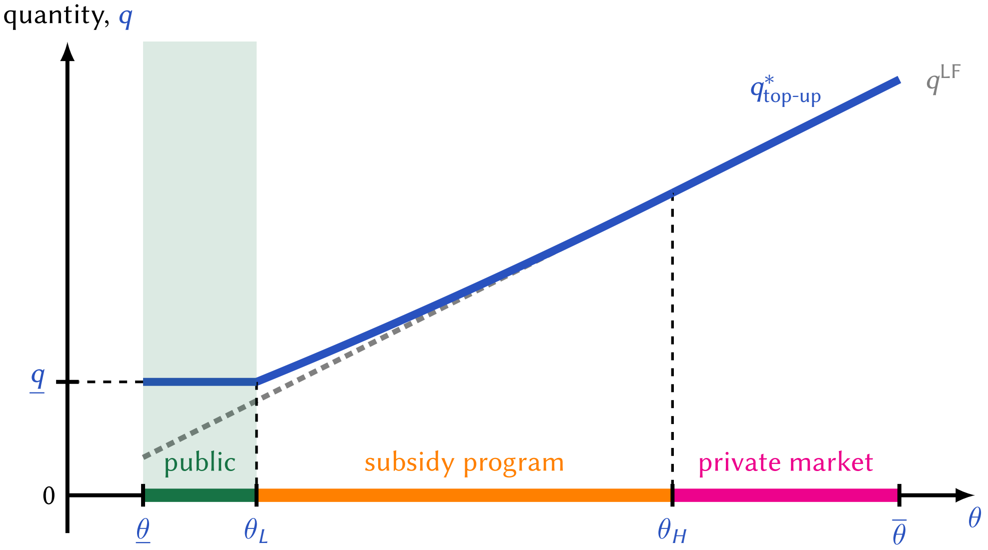

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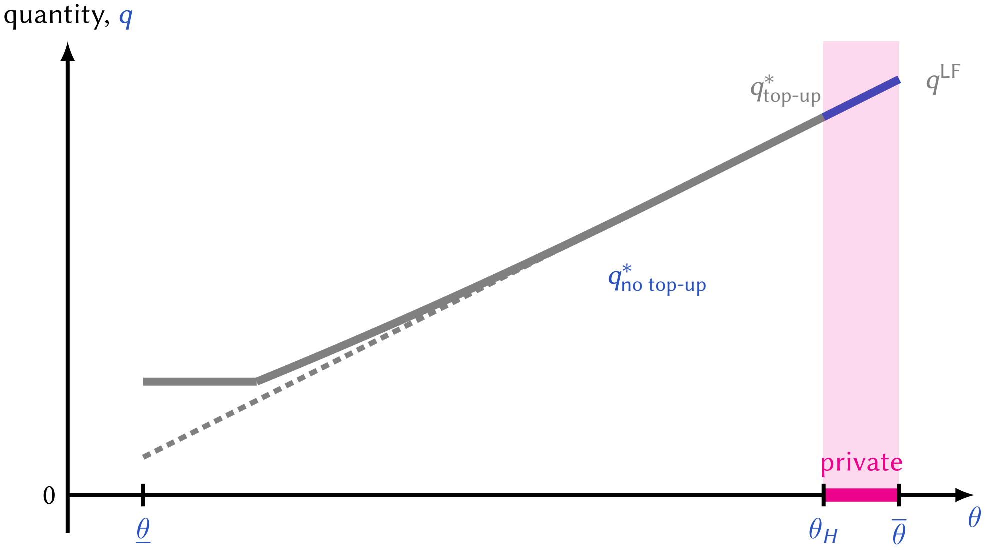

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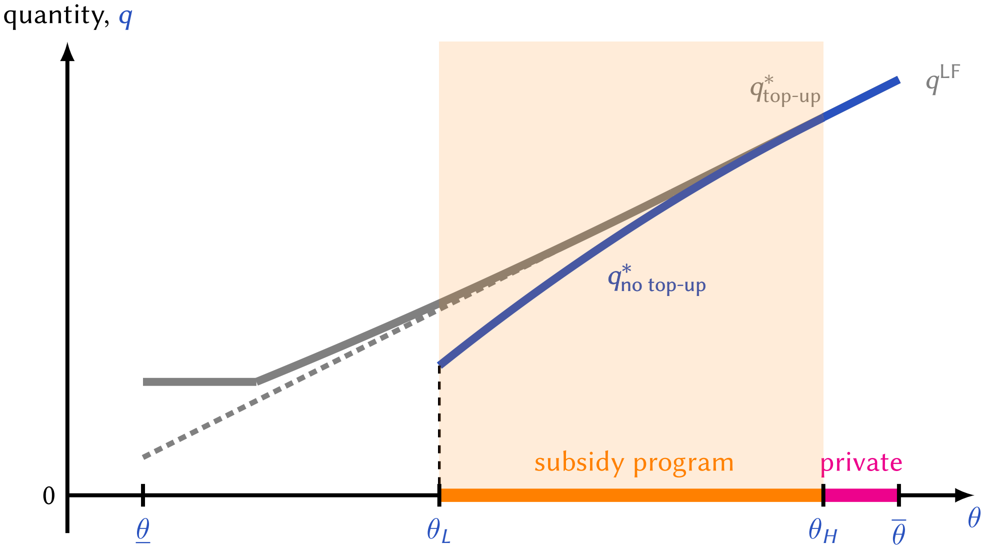

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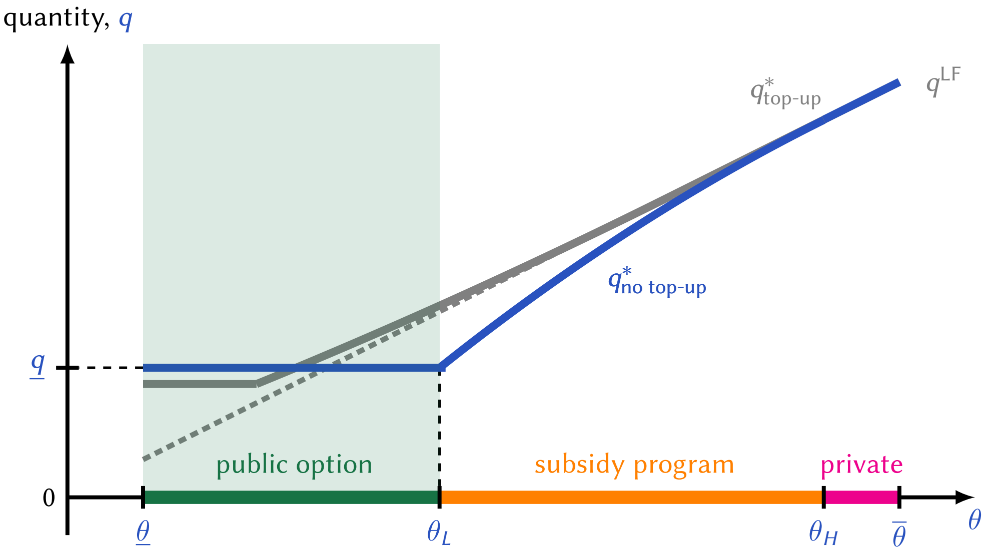

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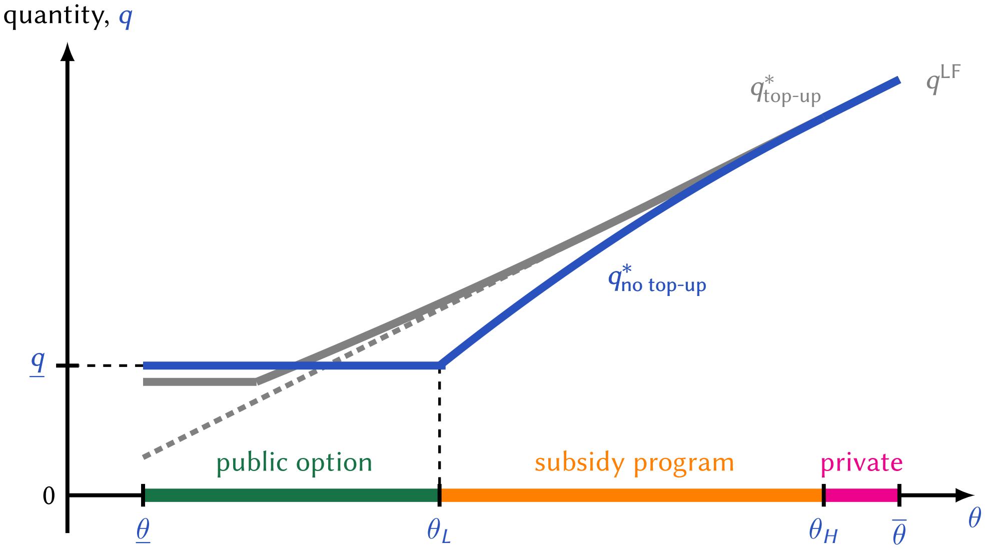
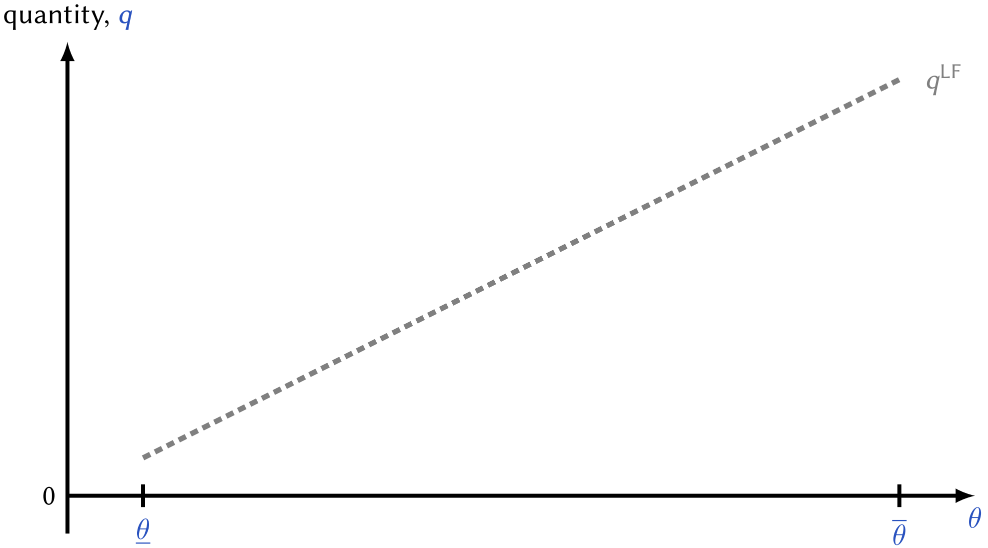
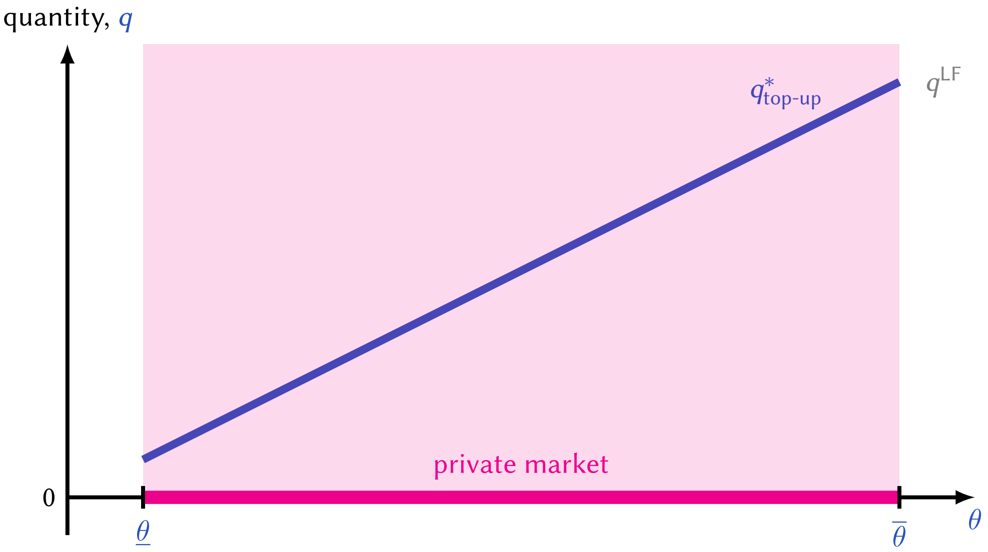


Characterization of Optimal Mechanisms: $\mathbf{E}[\omega] \leq \alpha < \max \omega$




quantity, q
$q^{*}_{\text{top-up}}$
q^{LF}
private market
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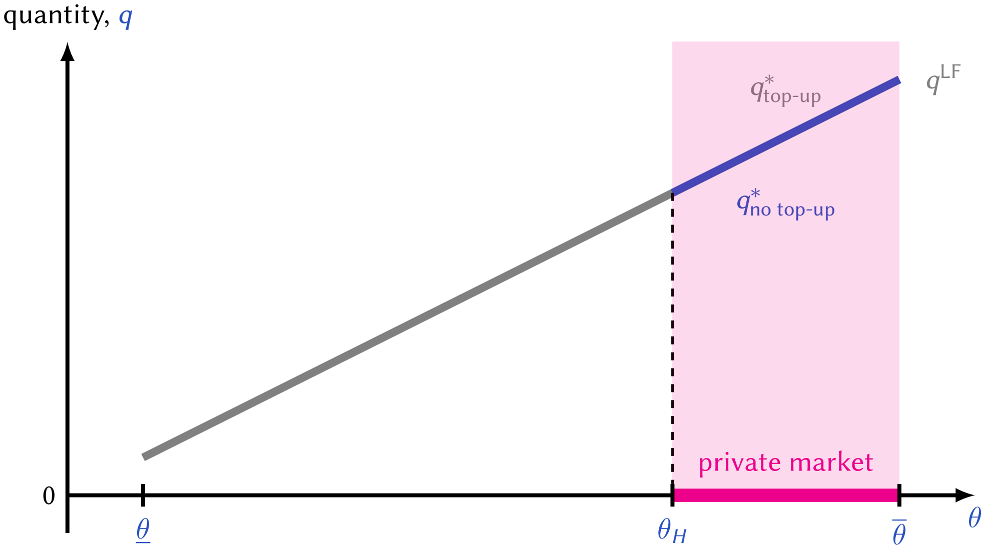

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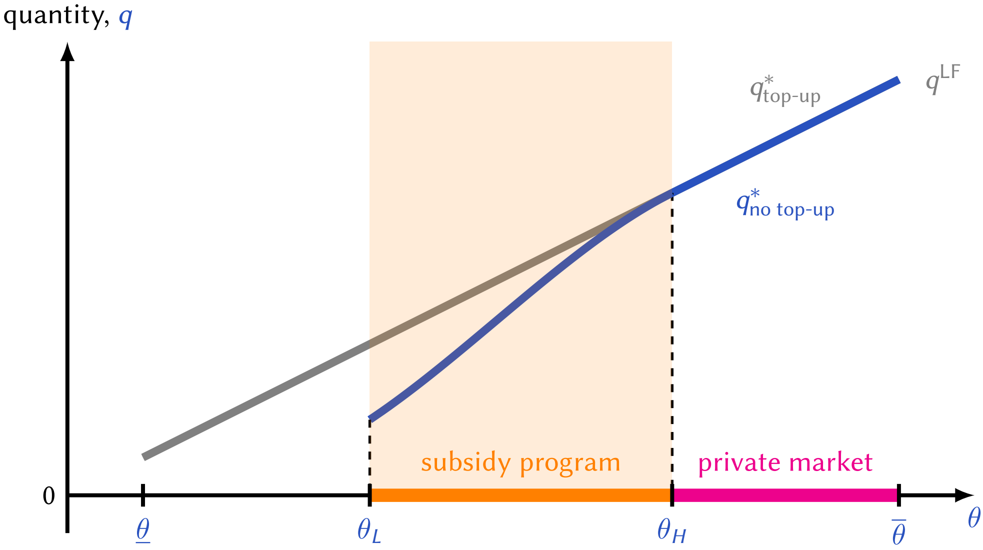

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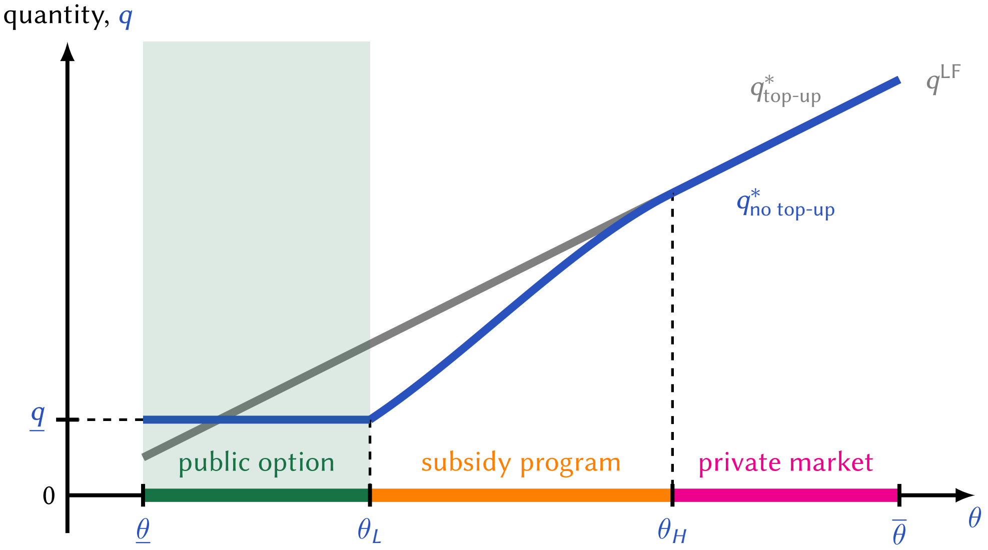

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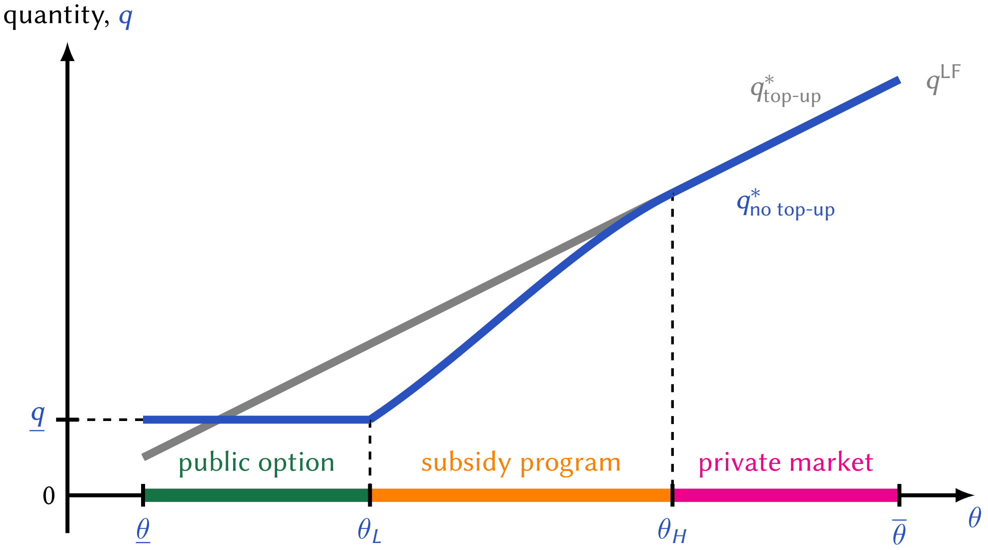

Theorem 4. When correlation is negative (i.e., ω is decreasing), there exists $\alpha_{\min} \in [\min \omega, \mathbf{E}[\omega])$ such that topping up **affects** optimal redistribution if and only if $\alpha_{\min} < \alpha < \max \omega$.

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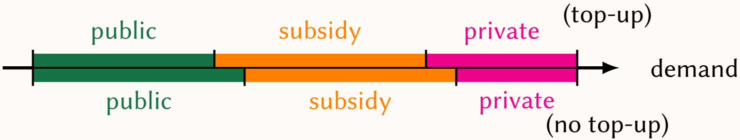

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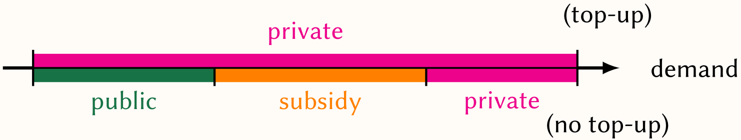

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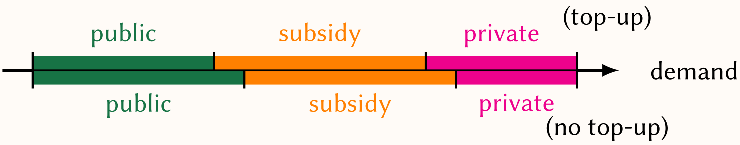

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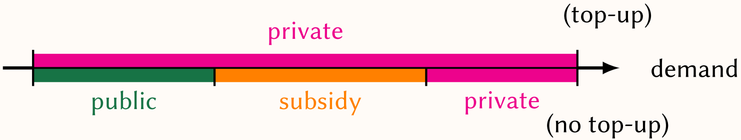


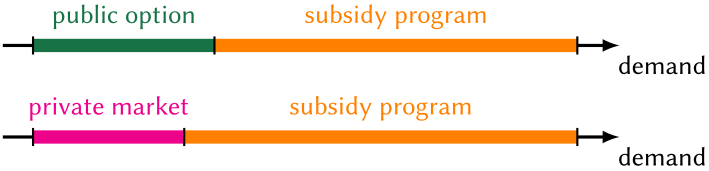
Topping up expands the use of the private market and reduces the use of a free public option.

Summary

1. Topping up reduces the social planner's scope for screening.
2. Topping up can distort screening even conditional on intervention.
3. When correlation is positive, optimal mechanisms coincide:

If $\mathbf{E}[\omega] > \alpha$:

If $\mathbf{E}[\omega] \leq \alpha$:



4. When correlation is negative, optimal mechanisms differ iff $\alpha \in (\alpha_{\min}, \max \omega)$:
   a. Topping up rotates consumption away from high priority consumers.
   b. Topping up expands the use of the private market.
   c. Topping up reduces the use of a free public option.

Discussion

Concluding Remarks

Redistribution programs often operate alongside (or within) private markets.

This leads to two challenges for optimal design:

- equilibrium effects; and
- (type-dependent) outside options.

This talk: how do these challenges affect optimal redistribution?

Future Work

#1. Imperfectly competitive private markets

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- **Equilibrium:** Mekonnen (2025); Mäkimattila (ongoing).
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#2. Instrument choice

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#2. Instrument choice

- **Simple instrument:** free public option.
- **Rich instruments:** nonlinear subsidies.
- **Something in between?**

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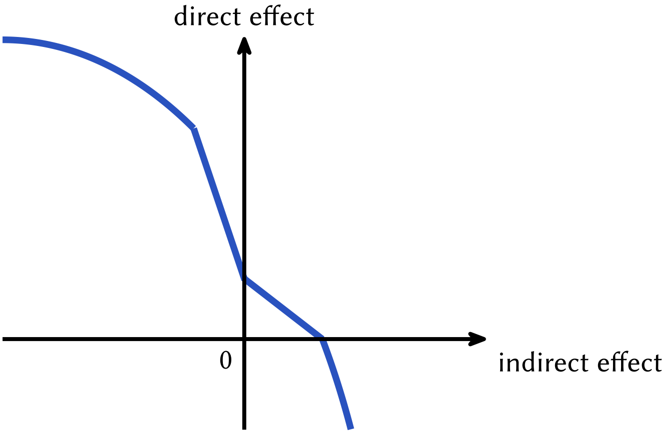
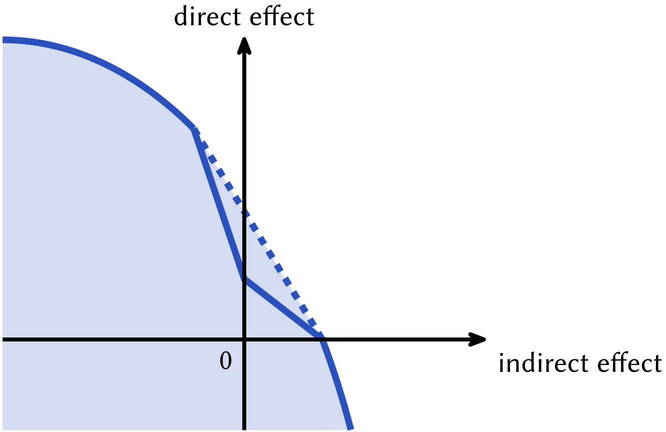
Nonconvexities in the Feasible Set

back



Randomizing between prices in the private market cannot help the social planner.



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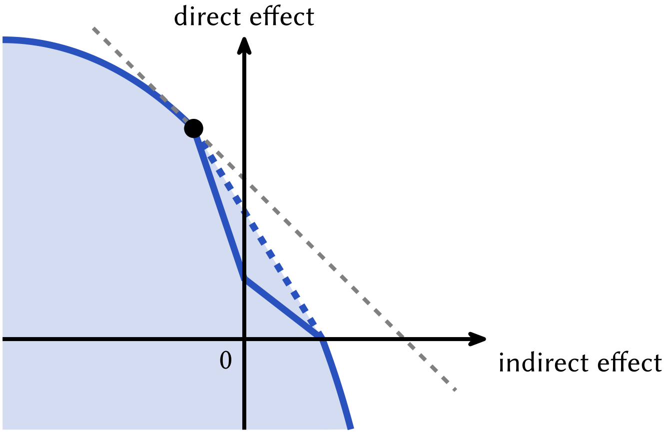

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