Optimal Indirect Regulation of Externalities

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- bans (*e.g.*, sale/manufacture of psychoactive drugs) and mandates (*e.g.*, vaccines);
- price restrictions (*e.g.*, minimum unit pricing laws for alcohol);
- quantity restrictions (*e.g.*, one-handgun-a-month laws).

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- quantity restrictions (*e.g.*, one-handgun-a-month laws).

Why don't policymakers set a Pigouvian tax/subsidy?

- It is often infeasible to measure how much externality each consumer generates.
- Instead, policymakers indirectly regulate the externality by regulating the good.

This paper

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#1. (*Policy evaluation*) How much deadweight loss does a given policy generate?

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This paper: develops approach that combines sufficient statistics + mechanism design.

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  - Consumption: derives utility $\theta v(q)$ from consuming $q \in [0, A]$ units of the good. Assume $v : [0, A] \to \mathbb{R}$ is non-decreasing and strictly concave; *e.g.*, $v(q) = Aq - \frac{1}{2}q^2$.

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- Assume consumer payoffs are additively separable in total externality:

$$\theta v(\underbrace{q(\theta, \xi)}_{\text{quantity consumed}}) - \underbrace{t(\theta, \xi)}_{\text{payment}} - \underbrace{E}_{\text{total externality}}, \quad \text{where } E = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\xi}}^{\overline{\xi}} \xi q(\theta', \xi') \, \mathrm{d}G(\theta', \xi').$$

First-best benchmark

- Suppose the externality ξ that each consumer produces can be measured.

  Then the FB outcome can be attained by setting a personalized **Pigouvian tax** of ξ.

- Under the Pigouvian tax, each consumer faces a marginal price of $c + \xi$ per unit.

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- But measuring and directly taxing the externality ξ is often infeasible in practice:
  - psychoactive drug use;
  - vaccination;
  - alcohol consumption;
  - gun purchase.

  Instead, policymakers indirectly regulate these externalities by taxing consumption.

Social planner's problem

- The social planner chooses a mechanism (q, t), consisting of:
  - an allocation function $q : [\underline{\theta}, \overline{\theta}] \times [\underline{\xi}, \overline{\xi}] \to [0, A]$; and
  - a payment function $t : [\underline{\theta}, \overline{\theta}] \times [\underline{\xi}, \overline{\xi}] \to \mathbb{R}$.

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- The social planner maximizes total surplus:

$$\mathsf{TS} = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\xi}}^{\overline{\xi}} [\theta v(q(\theta, \xi)) - (c + \xi) \cdot q(\theta, \xi)] \, \mathrm{d}G(\theta, \xi).$$

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- By the revelation principle, restrict attention WLOG to incentive-compatible (q, t):

$$(\theta, \xi) \in \underset{(\hat{\theta}, \hat{\xi})}{\arg\max} \left[\theta v(q(\hat{\theta}, \hat{\xi})) - t(\hat{\theta}, \hat{\xi}) - \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\xi}}^{\overline{\xi}} \xi' q(\theta', \xi') \, \mathrm{d}G(\theta', \xi') \right]. \tag{IC}$$

Given q, if there exists t such that (q, t) satisfies (IC), then q is implementable.

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Lemma 1. Define

$$\mathcal{Q} := \left\{ q : [\underline{\theta}, \overline{\theta}] \to [0, A] \text{ is non-decreasing} \right\}.$$

Then q is implementable only if there exists $\hat{q} \in \mathcal{Q}$ such that

$$q(\theta, \xi) = \hat{q}(\theta) \quad \text{for almost every } (\theta, \xi) \in [\underline{\theta}, \overline{\theta}] \times [\underline{\xi}, \overline{\xi}].$$

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- **Intuition:** consumer payoffs don't depend on reported ξ, cannot be truthfully elicited.
- **Implications:**
  - **#1.** Solution to social planner's problem does not depend on whether consumers observe ξ.
  - **#2.** Allows us to write q as function of only θ; $\mathcal{Q}$ is set of implementable allocation functions.

Questions

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#2. Given any subset $\mathcal{S} \subset \mathcal{Q}$ of implementable allocation functions, what is the allocation function $q^* \in \mathcal{S}$ in that set that minimizes deadweight loss?

(If $\mathcal{S} = \mathcal{Q}$, the optimal allocation function q^* is the second-best allocation function.)

Illustration

Illustration with linear demand

Assumption. $v(q) = Aq - \frac{1}{2}q^2$, where $c + \overline{\xi} < \underline{\theta}A$ so that $q^{\text{FB}} \in (0, A)$ is interior.

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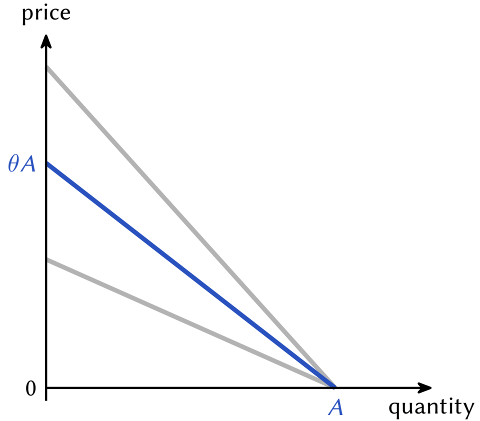
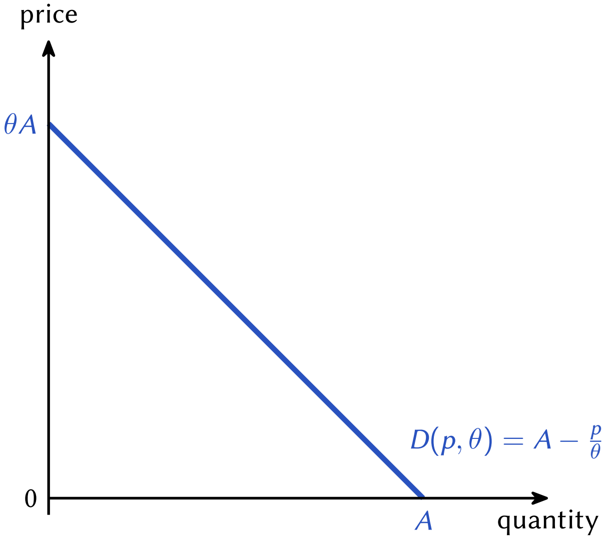
- Each consumer has an individual demand curve given by

$$D(p, \theta) = A - \frac{p}{\theta}.$$

- Simple way to capture continuous demand for homogeneous good.
- Can be viewed as a local approx. in the spirit of Harberger (1964).



Deadweight loss



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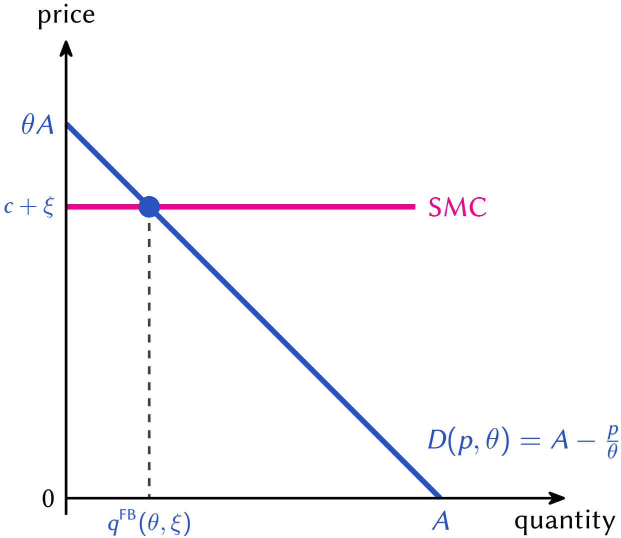

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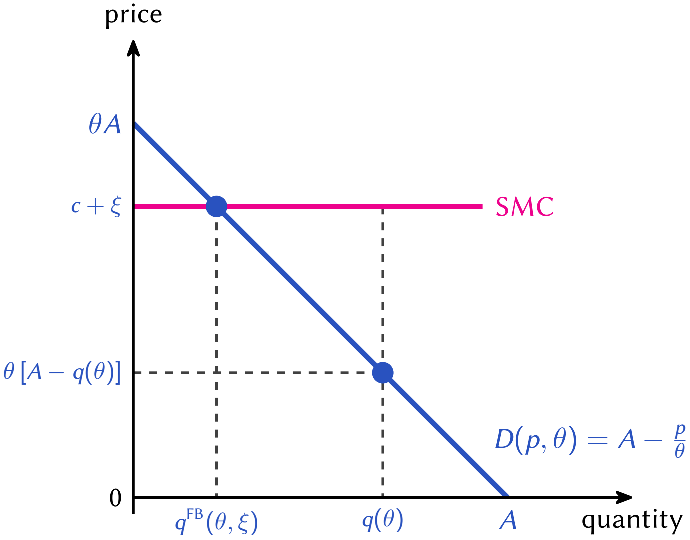

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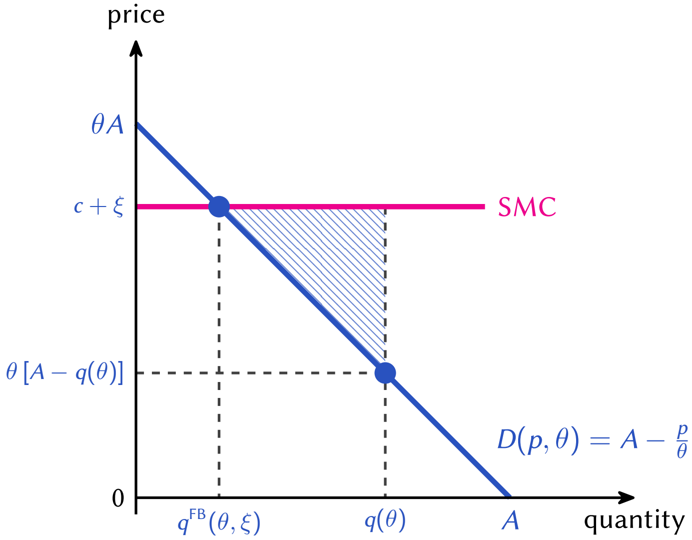

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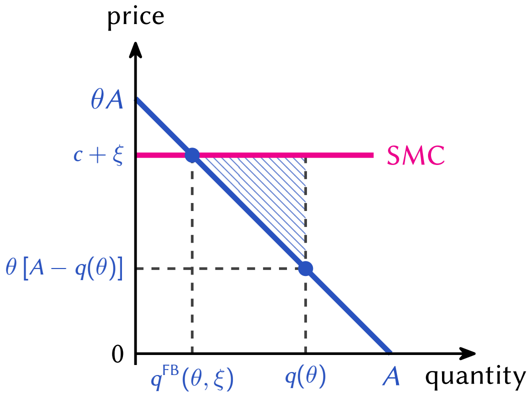

$$\mathrm{DWL}(\theta,\xi)=\frac{1}{2}\times\left[q^{\mathrm{FB}}(\theta,\xi)-q(\theta,\xi)\right]\times\theta\left[q^{\mathrm{FB}}(\theta,\xi)-q(\theta,\xi)\right]=\frac{\theta}{2}\left[q^{\mathrm{FB}}(\theta,\xi)-q(\theta,\xi)\right]^2.$$

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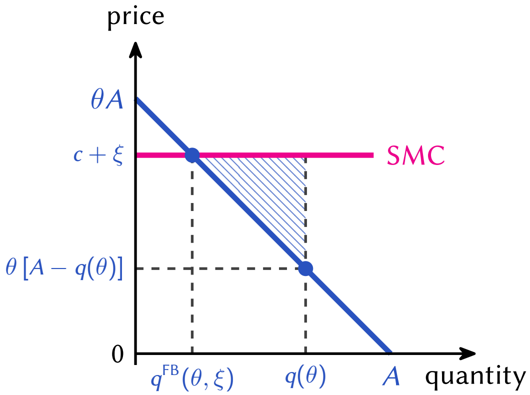

Proposition 1. For any incentive-compatible mechanism (q, t), the deadweight loss is equal to

$$\mathrm{DWL} = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\xi}}^{\overline{\xi}} \frac{\theta}{2}\left[q^{\mathrm{FB}}(\theta,\xi) - q(\theta)\right]^2 \,\mathrm{d}G(\theta,\xi).$$

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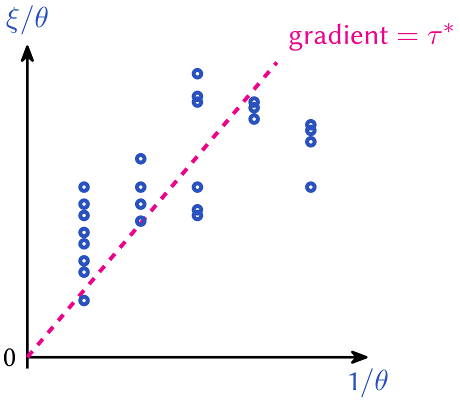

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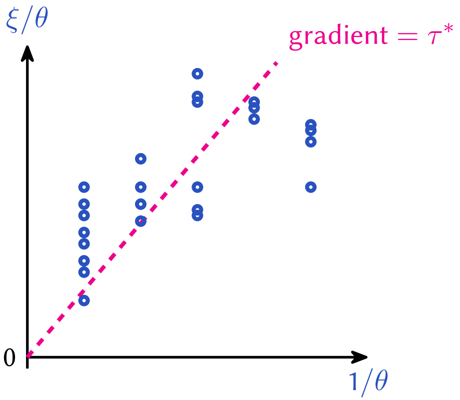
#1. SSR (weighted by θ)

$$= \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\xi}}^{\overline{\xi}} \theta \left[q^{\mathrm{FB}}(\theta,\xi) - q(\theta,\xi) \right]^2 \, \mathrm{d}G(\theta,\xi)$$

$$= 2 \times \mathrm{DWL}.$$



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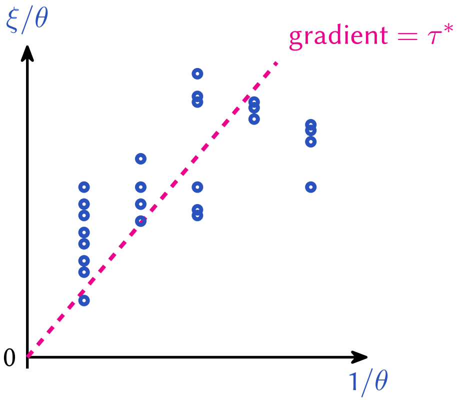
#2. The optimal linear tax is $\tau^* = \dfrac{\mathbf{E}[\xi/\theta]}{\mathbf{E}[1/\theta]}$.

$\rightsquigarrow$ Cf. Diamond (1973).



What if we allow nonlinear taxes?

The second-best allocation function q^{SB} is obtained by regressing q^{FB} on $\mathcal{Q}$:

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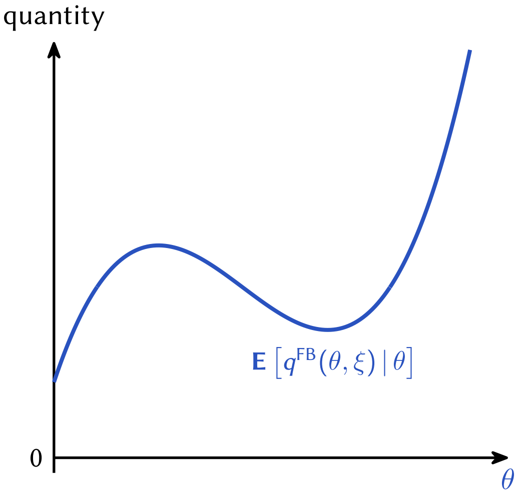
Recall that $\mathcal{Q} := \left\{ q : [\underline{\theta}, \overline{\theta}] \to [0, A] \text{ is non-decreasing} \right\}$.

This means that q^{SB} is the **isotonic regression** of q^{FB} on θ.

Second-best allocation



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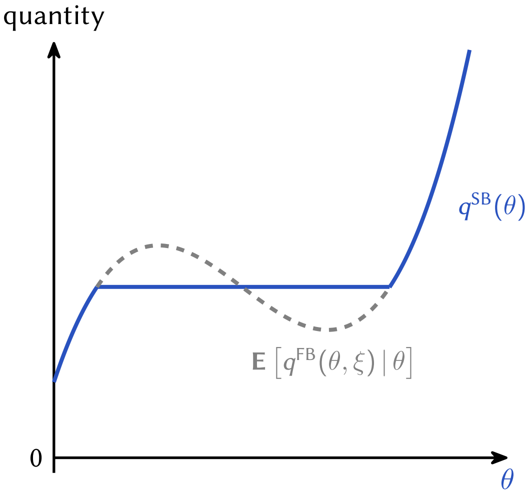

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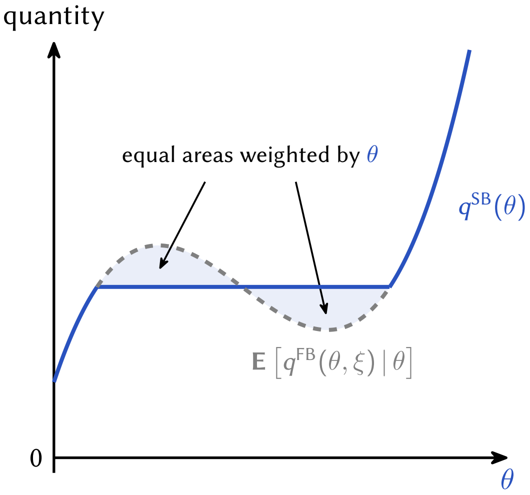
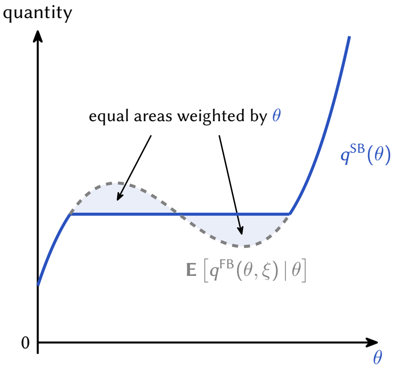


Second-best allocation

Proposition 2. There is a unique optimal allocation function q^{SB} given by

$$q^{\mathrm{SB}}(\theta) = \frac{\mathrm{d}}{\mathrm{d}s}\left(\mathrm{co}\int_{1-s}^{1} \mathbf{E}\left[q^{\mathrm{FB}}(\hat{\theta},\xi)\,|\,\hat{\theta} = W^{-1}(z)\right]\,\mathrm{d}z\right)\Bigg|_{s=1-W(\theta)},\; W(\theta) = \frac{1}{\mathbf{E}[\theta]}\int_{\underline{\theta}}^{\theta}\int_{\underline{\xi}}^{\overline{\xi}} zg(z,\xi)\,\mathrm{d}z\,\mathrm{d}\xi.$$



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Although construction of q^{SB} uses ironing (Myerson, 1981), it is different from other problems:

- In other problems, the MR curve (or equivalent) is ironed.
- Here, the (expected) first-best allocation function $\mathbf{E}[q^{\text{FB}}(\theta,\xi) \mid \theta]$ is being ironed.

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Proof idea: By Proposition 1, q^{SB} = **WLS projection** (with weights equal to θ) of q^{FB} onto $\mathcal{Q}$.

Technical step in paper: WLS projection operator is given by q^{SB} in statement of Proposition 2.

Takeaways from the case of linear demand

#1. What is the deadweight loss of any incentive-compatible mechanism (q, t)?

#2. Given any subset $\mathcal{S} \subset \mathcal{Q}$ of implementable allocation functions, what is the allocation function $q^* \in \mathcal{S}$ in that set that minimizes deadweight loss?

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The rest of this paper shows that regression approach also works for general demand, by: *(i)* generalizing regression loss function and *(ii)* characterizing resulting projection.

Conclusion

- This paper develops a **regression approach** to indirectly regulate externalities.
  - Deadweight loss is equal to the residual from the regression (*i.e.*, regression loss).
  - Optimal indirect policy obtained by characterizing projection associated with regression.
- The results of this paper also...
  - show that "non-market" policies, such as price and quantity controls, can be optimal;
  - show how to implement allocations (nonlinear taxes can be derived via regression); and
  - have empirical relevance (*e.g.*, regulation of automobile externalities via VMT taxation).

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Thank you!

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