Prices vs. Quantities: Robust Regulation

Zi Yang Kang

University of Toronto

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Motivation

Policymakers frequently rely on **quantity controls** to **regulate externalities**.

- **Bans and mandates**: recreational drugs, immunizations.
- **Floors and ceilings**: minimum education, one-gun-a-month laws.

Yet, many economists believe in the superiority of **price regulations** (taxes and subsidies).

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COVID-19 pandemic:

- Many economists advocated for paying individuals to get vaccinated.
- Many schools, employers, and governments instead chose to impose vaccine mandates.

Prices vs. Quantities

This may seem like the "prices-versus-quantities" problem studied by Weitzman (1974)...

- **Rich information**: regulator knows full distribution of externalities (Bayesian uncertainty).
- **Simple instruments**: regulator has limited instruments (either fixed quantity or fixed price).

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...but it is different.

- **Simple information**: regulator knows only average externality (non-Bayesian uncertainty).
- **Rich instruments**: regulator has no restriction on instruments (uses full mechanism design).

This paper: What is the **optimal regulation** with **simple information** and **rich instruments**?

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Main result: the **optimal policy** is a **quantity control** (*positive*: floor, *negative*: ceiling).

Applications: vaccines (goods with unit demand), regulator uncertainty, costly screening.

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- Agents are heterogeneous in two dimensions:

  #1. Consumption—type $\theta \in \Theta \subset \mathbb{R}_+$ determines utility from consuming q units, $u(q, \theta)$.

  Assumptions: Θ compact; $\theta \sim F \in C^1$; $F' > 0$ on Θ; $u_q > 0$, $u_{qq} < 0$, $u_{q\theta} > 0$ for $q \in [0, A]$.

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- Utility depends on own consumption $q(\theta, \xi)$, payment $t(\theta, \xi)$, aggregate externality E:

$$u(q(\theta,\xi),\theta) - t(\theta,\xi) + E, \qquad \text{where } E = \iint_{(\hat{\theta},\hat{\xi}) \in \Theta \times \mathbb{R}_+} \hat{\xi} q(\hat{\theta},\hat{\xi}) \, \mathrm{d}H(\hat{\theta},\hat{\xi}).$$

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  Let $m(\theta) := \mathbf{E}_H[\xi \mid \theta]$ and consider three informational benchmarks:

  #1. unknown correlation—regulator considers any $m : \Theta \longrightarrow \mathbb{R}_+$ such that $\mu = \mathbf{E}_F[m(\theta)]$;

  #2. positive correlation—regulator considers #1. and also requires m to be nondecreasing;

  #3. negative correlation—regulator considers #1. and also requires m to be nonincreasing.

  Denote the respective corresponding feasible sets of $m : \Theta \longrightarrow \mathbb{R}_+$ by $\mathcal{M}_0$, $\mathcal{M}_+$, and $\mathcal{M}_-$.

Mechanism Design

- By the revelation principle, restrict attention to **incentive-compatible** (q, t) WLOG:

$$(\theta, \xi) \in \arg\max_{(\hat{\theta}, \hat{\xi})} \left[u(q(\hat{\theta}, \hat{\xi}), \theta) - t(\hat{\theta}, \hat{\xi}) + E \right]. \tag{IC}$$

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- By (IC), all agents of same type θ reports the same $\hat{\xi}$ (or are indifferent over multiple).

  Following the logic of Myerson (1981), can show that (q, t) satisfies (IC) for some t iff

$$q(\hat{\theta}, \hat{\xi}) \overset{a.e.}{=} \hat{q}(\hat{\theta}), \qquad \text{where } \hat{q} : \Theta \to [0, A] \text{ is nondecreasing.}$$

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Abusing notation, consider $q : \Theta \to [0, A]$ and $t : \Theta \to \mathbb{R}$ as functions of just type.

Denote the feasible set of all such q by Q (the set of "implementable allocations").

Robust Mechanism Design

- The regulator evaluates policies by their **worst-case welfare performance**.
- If the joint distribution H were known, then total surplus would be

$$\int_{\Theta} [u(q(\theta), \theta) + [m(\theta) - c]\, q(\theta)]\ \mathrm{d}F(\theta).$$

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- Under the three informational benchmarks $* \in \{0, +, -\}$, the regulator solves

$$\sup_{q \in \mathcal{Q}} \inf_{m \in \mathcal{M}_*} \int_{\Theta} \left[u(q(\theta), \theta) + [m(\theta) - c]\, q(\theta) \right] \, \mathrm{d}F(\theta);$$

#1. unknown correlation: $\mathcal{M}_0 := \{m : \Theta \to \mathbb{R}_+ \text{ is measurable and } \mu = \mathbf{E}_F[m(\theta)]\}$;

#2. positive correlation: $\mathcal{M}_+ := \{m : \Theta \to \mathbb{R}_+ \text{ is nondecreasing and } \mu = \mathbf{E}_F[m(\theta)]\}$;

#3. negative correlation: $\mathcal{M}_- := \{m : \Theta \to \mathbb{R}_+ \text{ is nonincreasing and } \mu = \mathbf{E}_F[m(\theta)]\}$.

Main Results

Main Result #1: Unknown Correlation

Theorem 1. Let q^{LF} denote the laissez-faire allocation function. Under the unknown-correlation benchmark, there exists a **quantity floor** $\underline{q} \in [0, A]$ such that the **unique** optimal allocation is

$$q_0^*(\theta) = \max\left\{q^{\mathsf{LF}}(\theta), \underline{q}\right\}.$$

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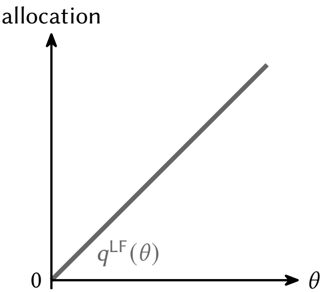

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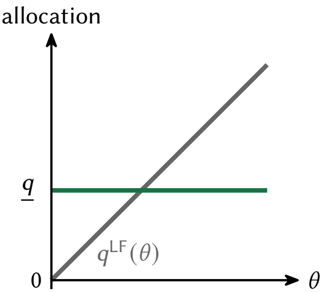

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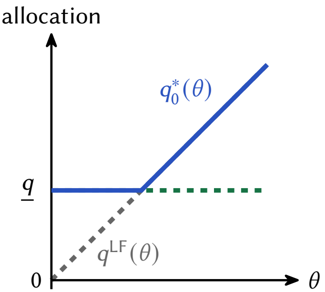

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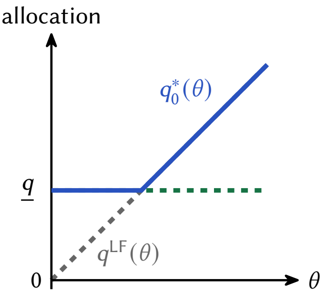

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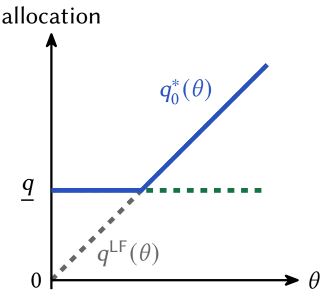

#1. There is a **quantity floor** of $\underline{q}$.

This maximally lower-bounds MB generated, subject to IC.

#2. There is **no intervention** above $\underline{q}$.

This is different from Bayesian mechanism design!

Intervention leads to unnecessary distortion in worst case.

Main Result #2: Known Correlation

Theorem 2. Let $D(p, \theta)$ be the demand function of an agent of type θ. Under the positive-correlation benchmark, the **unique** optimal allocation can be implemented by a **constant per-unit subsidy equal to μ**:

$$q_+^*(\theta) = D(c - \mu, \theta).$$

Under the negative-correlation benchmark, the unique optimal allocation is $q_-^*(\theta) = q_0^*(\theta)$.

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- This is again different from Bayesian mechanism design!

  In fact, Diamond (1973) and subsequent authors caution against this “naïve” subsidy.

Discussion

Returning to the motivating example of vaccines, consider $u(q,\theta) = \theta q$, with $q \in [0, q]$.

Proposition 1. Under the unknown-correlation benchmark and negative-correlation benchmark, the optimal allocation is

$$q_0^*(\theta) = q_-^*(\theta) = \begin{cases} 1, & \text{if } \mu \geq \mathbf{E}_F[(c-\theta)_+], \\ q^{\mathrm{LF}}(\theta) = \mathbf{1}_{\theta > c}, & \text{if } \mu < \mathbf{E}_F[(c-\theta)_+]. \end{cases}$$

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- By **Proposition 1**, there are three candidates: **mandate**, **no intervention**, **subsidy**.
  - **Mandate** vs. **no intervention**: avg. benefit μ vs. avg. DWL from mandate $\mathbf{E}_F[(c-\theta)_+]$.
  - **Subsidy**: greater risk of exposure $\rightsquigarrow$ higher demand for vaccination (e.g., HPV, mpox).

Other Results

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- Results extend to the case of **regulatory uncertainty** *à la* Weitzman (1974).

  Correlation direction between cost and benefit shifters determines optimal instrument.

- Results extend to the case of **costly screening** (e.g., for redistribution).

  Robustness against correlation between MRS and waiting costs $\rightsquigarrow$ optimal rationing.

  This complements a growing literature on Bayesian redistributive mechanism design.

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As a subsequent literature shows, richer price instruments can dominate quantity controls.

This paper differs along both dimensions: **rich instruments**, **simple information**.

- Results provide a new explanation for prevalence of quantity controls: **robust optimality**.
- With information about direction of correlation, **simple price regulations** can be optimal.

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