Prices vs. Quantities: Robust Regulation

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COVID-19 pandemic:

- Many economists advocated paying individuals to get vaccinated.
- Many schools, employers, and governments instead imposed vaccine mandates.

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- **Rich information**: regulator knows full distribution of externalities (Bayesian uncertainty).
- **Simple instruments**: regulator has limited instruments (either fixed quantity or fixed price).

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…but it is different.

- **Simple information**: regulator knows only average externality (non-Bayesian uncertainty).
- **Rich instruments**: regulator designs a nonlinear pricing schedule (full mechanism design).

This paper: What is the **optimal regulation** with **simple information** and **rich instruments**?

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Role for prices: information about selection can justify **price regulation** (tax or subsidy).

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  #1. Consumption—type $\theta \in \Theta := \prod_{i=1}^{d} [\underline{\theta}_i, \overline{\theta}_i]$ determines direct utility from q units, $u(q, \theta)$.

  Assumptions: $\theta \sim F \in C^1$ with positive density f on Θ; $u_q > 0$, $u_{qq} < 0$ for $q \in [0, A]$;
  u has strict increasing differences in (q, θ)—e.g., $u_{q\theta_i} > 0$ for $i \in \{1, \ldots, d\}$.

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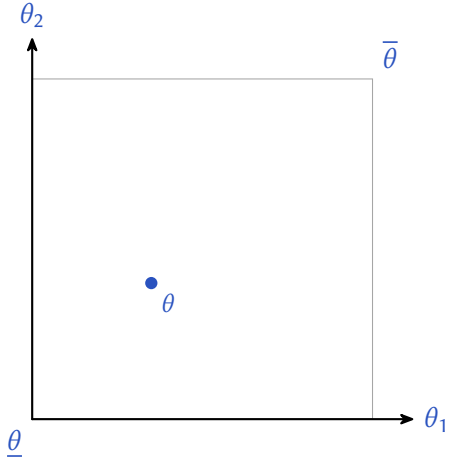
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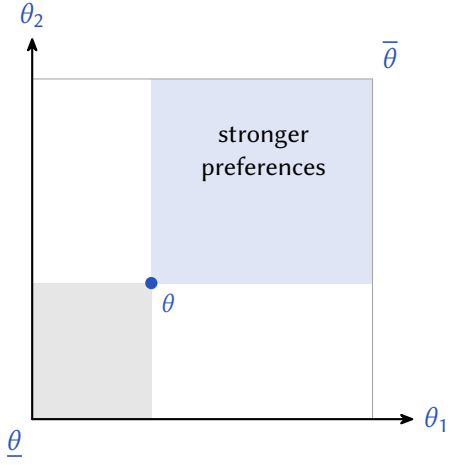
- Utility from own consumption $q(\theta, \xi)$, payment $t(\theta, \xi)$, aggregate externality $E(q)$:

$$u(q(\theta, \xi), \theta) - t(\theta, \xi) + E(q), \qquad \text{where } E(q) = \iint_{(\hat{\theta}, \hat{\xi}) \in \Theta \times \mathbb{R}_+} \hat{\xi} q(\hat{\theta}, \hat{\xi}) \, \mathrm{d}H(\hat{\theta}, \hat{\xi}).$$

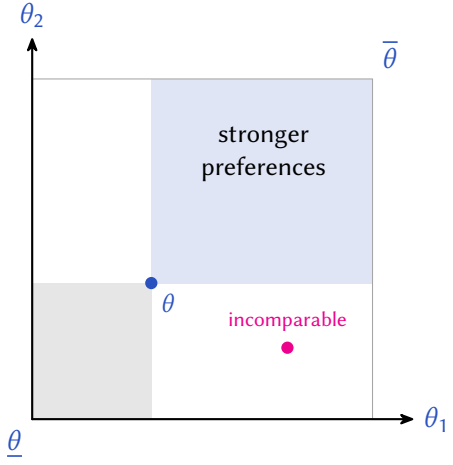
Example: Two-Dimensional Consumption Preferences



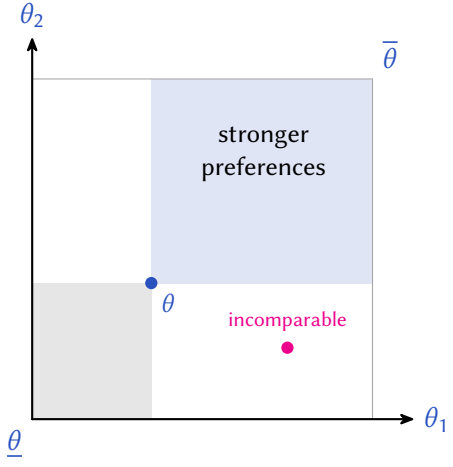
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Coordinates of θ may be correlated with each other and with ξ.

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  Examples:

  - **price regulation**: $T(q) = (c - s)\, q$ implements a uniform subsidy s;
  - **quantity regulation**: $T(q) = cq\mathbf{1}_{q \geq \underline{q}} + \infty \mathbf{1}_{q < \underline{q}}$ implements a quantity floor $\underline{q}$.

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- Equivalently (Myerson, 1979), she chooses a direct mechanism (q, t), consisting of:
  - the **allocation function** $q : \Theta \times \mathbb{R}_+ \longrightarrow [0, A]$ denoting consumption by each agent;
  - the **payment function** $t : \Theta \times \mathbb{R}_+ \longrightarrow \mathbb{R}$ denoting payment by each agent.

Mechanism Design

- By the revelation principle, restrict attention to **incentive-compatible** (q, t) WLOG:

$$(\theta, \xi) \in \arg\max_{(\hat{\theta}, \hat{\xi})} \left[u(q(\hat{\theta}, \hat{\xi}), \theta) - t(\hat{\theta}, \hat{\xi}) + E(q) \right]. \tag{IC}$$

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  Following Myerson (1981), one can show that (q, t) satisfies (IC) for some t only if

$$q(\hat{\theta}, \hat{\xi}) \overset{a.e.}{=} \hat{q}(\hat{\theta}), \qquad \text{where } \hat{q} \in Q := \{\hat{q} : \Theta \to [0, A] \text{ is nondecreasing}\}.$$

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- Henceforth, consider the **relaxed** feasible set Q (will show solution is implementable).

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  #2. positive selection—regulator considers **#1.** and also requires m to be "nondecreasing";

  #3. negative selection—regulator considers **#1.** and also requires m to be "nonincreasing."

  Denote the respective corresponding feasible sets of $m : \Theta \to \mathbb{R}_+$ by $\mathcal{M}_0$, $\mathcal{M}_+$, and $\mathcal{M}_-$.

Positive and Negative Selection

Let $m(\theta) \coloneqq \mathbf{E}_H[\xi \mid \theta]$ and $\mu \coloneqq \mathbf{E}_F[m(\theta)]$. For any measurable set of types $B \subset \Theta$, define

$$F(B) \coloneqq \int_B \mathrm{d}F(\theta) \quad \text{and} \quad F^m(B) \coloneqq \frac{1}{\mu} \int_B m(\theta)\, \mathrm{d}F(\theta).$$

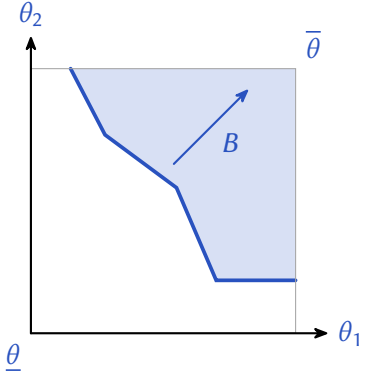
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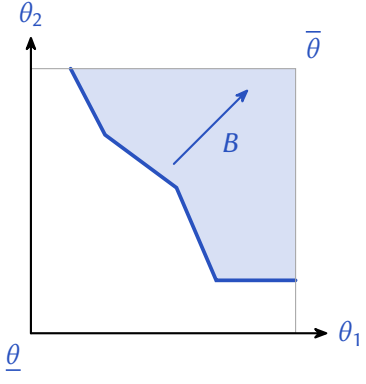


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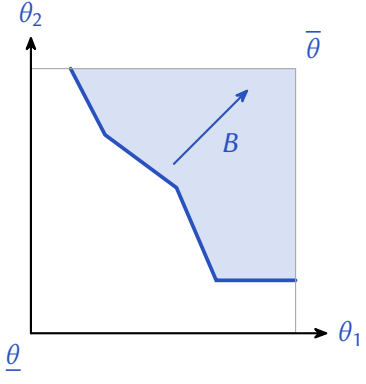
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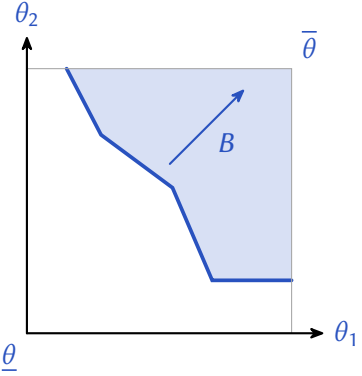
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Robust Mechanism Design

- The regulator evaluates policies by their **worst-case welfare performance**.
- If the joint distribution H were known, then total surplus would be

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- Under the three informational benchmarks $* \in \{0, +, -\}$, the regulator solves

$$\sup_{q \text{ is implementable}} \inf_{m \in \mathcal{M}_*} \int_{\Theta} \left[u(q(\theta), \theta) + \left[m(\theta) - c \right] q(\theta) \right] \, \mathrm{d}F(\theta);$$

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Main Results

Main Result #1: Unknown Selection

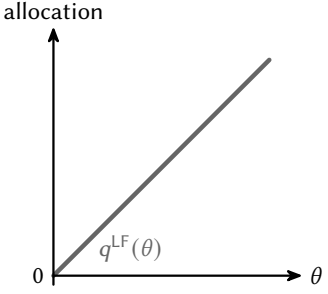
Theorem 1. Let q^{LF} denote the laissez-faire allocation function. Under unknown selection, there exists a **quantity floor** $\underline{q} \in [0, A]$ such that the **unique** optimal allocation is

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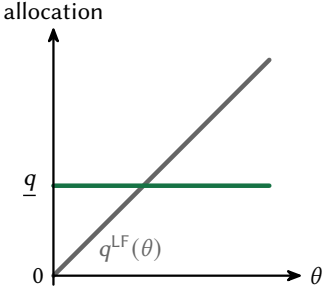
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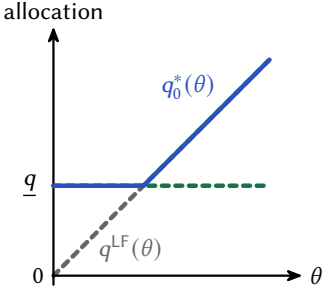
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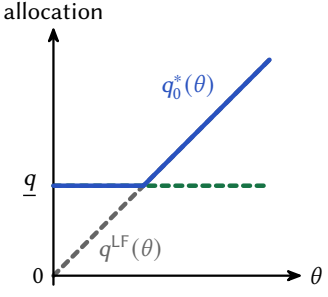
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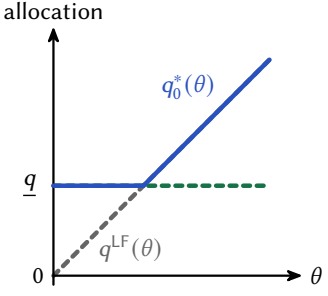
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#1. There is a **quantity floor** of $\underline{q}$.

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#2. There is **no intervention** above $\underline{q}$.

Intervention leads to unnecessary distortion in worst case.

Remark: Choosing the Optimal Floor

Raising the floor changes consumption only where $q^{\mathrm{LF}}(\theta) < \underline{q}$.

An interior optimum therefore satisfies

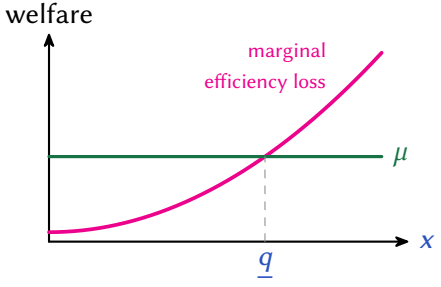
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This is **one-dimensional optimization**, even with multidimensional preferences.

- F affects the level of the floor through aggregate efficiency losses.
- Greater μ raises the optimal floor.

Main Result #2: Known Selection

Theorem 2. Let $D(p, \theta)$ be the demand function of an agent of type θ.

Under positive selection, the **unique** optimal allocation can be implemented by a **uniform per-unit subsidy equal to** μ:

$$q_+^*(\theta) = D(c - \mu, \theta).$$

Under negative selection, the unique optimal allocation is $q_-^*(\theta) = q_0^*(\theta)$.

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- The worst case under negative selection coincides with unknown selection.

  It remains to show that a uniform subsidy is optimal under positive selection.

Summary: What Information Changes

selection	worst-case external benefit	optimal instrument
unknown	$\mu \operatorname{ess\,inf}_{\theta \in \Theta} q(\theta)$	quantity floor
negative	$\mu \operatorname{ess\,inf}_{\theta \in \Theta} q(\theta)$	quantity floor
positive	$\mu \mathbf{E}_F[q(\theta)]$	uniform subsidy

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- Robustness makes a simple statistic of consumption sufficient for external benefit.
- With known heterogeneous m, even the optimal uniform subsidy is generally a weighted average (by demand response), rather than μ.

  In fact, Diamond (1973) and subsequent authors caution against this "naïve" subsidy.

Prices vs. Quantities

	positive externality	negative externality
unknown selection	quantity floor	quantity ceiling
positive selection	uniform subsidy	quantity ceiling
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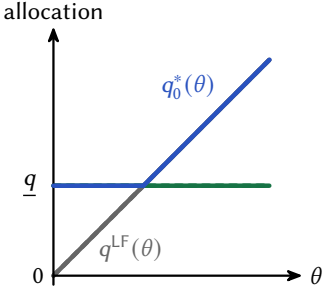
Price regulation: optimal when information on selection rules out that concentration.

Proofs of Main Results

Main Result #1: Unknown Selection

Theorem 1. Let q^{LF} denote the laissez-faire allocation function. Under unknown selection, there exists a **quantity floor** $\underline{q} \in [0, A]$ such that the **unique** optimal allocation is

$$q_0^*(\theta) = \max\left\{q^{\mathsf{LF}}(\theta), \underline{q}\right\}.$$



#1. There is a **quantity floor** of $\underline{q}$.

This maximally lower-bounds MB generated, subject to IC.

#2. There is **no intervention** above $\underline{q}$.

Intervention leads to unnecessary distortion in the worst case.

Proof Overview

The proof of Theorem 1 proceeds in three steps:

#1. Solving Nature's problem.

Nature concentrates marginal benefit among types consuming the least.

Given any allocation function q, the worst-case external benefit is $\mu \operatorname{ess\,inf}_{\theta \in \Theta} q(\theta)$.

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#3. Showing implementability.

The allocation $q_0^*(\theta) = \max\{q^{\mathrm{LF}}(\theta), \underline{q}\}$ can be implemented by a quantity floor.

Step #1: Solving Nature's Problem

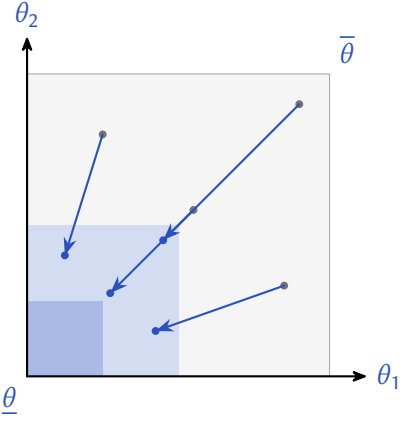
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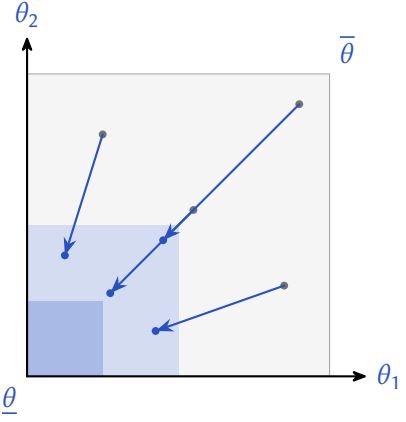
$$\phi_n : \theta \mapsto \underline{\theta} + \frac{\theta - \underline{\theta}}{n}.$$

This preserves the mean externality μ.

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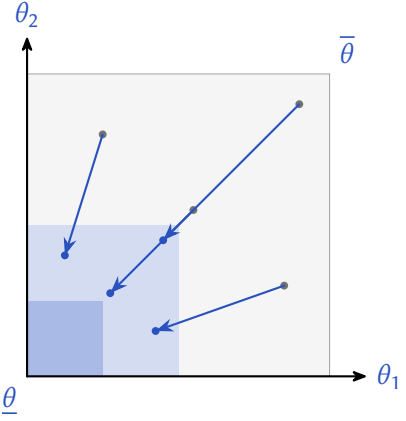
The external benefit is then

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So $\inf_{m \in \mathcal{M}_0} \mathbf{E}_F[m(\theta)q(\theta)] = \mu \operatorname{ess\,inf}_{\theta \in \Theta} q(\theta)$.

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Given Nature's worst-case external benefit, the regulator's relaxed problem becomes

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For each $\underline{q} \in [0, A]$ and $\theta \in \Theta$, strict concavity implies

$$\arg\max_{q \in [\underline{q}, A]} [u(q, \theta) - cq] = \max\left\{ D(c, \theta), \underline{q} \right\}.$$

This pointwise maximizer is nondecreasing, so it belongs to Q.

Step #3: Showing Implementability

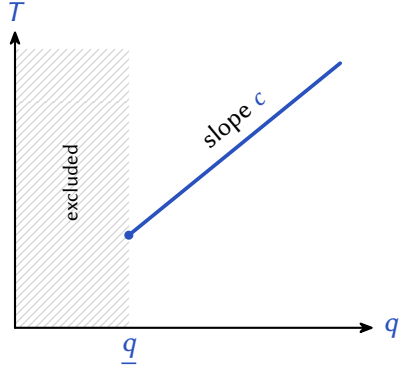
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Every type chooses

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The relaxed optimum is **implementable**.

The optimal allocation is $q_0^*(\theta) = \max\left\{q^{\mathrm{LF}}(\theta), \underline{q}\right\}$.

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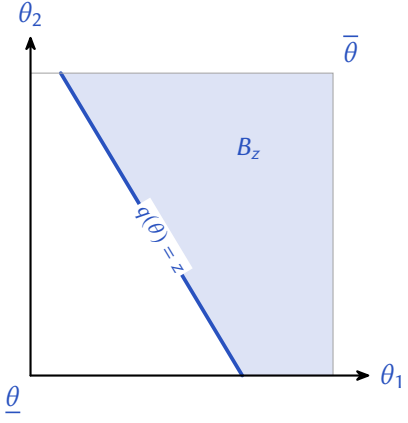
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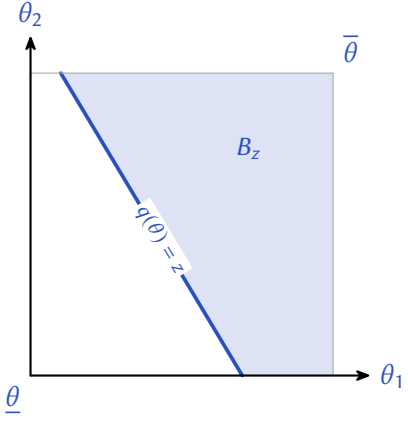


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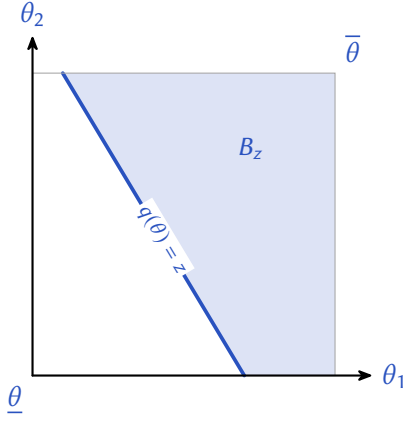
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Equality is attained by **no selection**: $m(\theta) \equiv \mu$. Thus, $\inf_{m \in \mathcal{M}_+} \mathbf{E}_F[m(\theta)q(\theta)] = \mu\,\mathbf{E}_F[q(\theta)]$.

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Remark: The optimal subsidy is μ and does not require information about F.

Discussion

Extension #1: Unit Demand

Consider linear utility $u(q, \theta) = v(\theta)q$ for some increasing $v : \Theta \to \mathbb{R}$, with $q \in [0, 1]$.

Proposition 1. Under unknown or negative selection, an optimal allocation is

$$q_0^*(\theta) = q_-^*(\theta) = \begin{cases} 1, & \text{if } \mu \geq \mathbf{E}_F[[c - v(\theta)]_+], \\ q^{\mathsf{LF}}(\theta) = \mathbf{1}_{v(\theta) > c}, & \text{if } \mu < \mathbf{E}_F[[c - v(\theta)]_+]. \end{cases}$$

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- By Proposition 1, there are three candidates: **mandate**, **no intervention**, **subsidy**.
  - Under unknown or negative selection, either a **mandate** or **no intervention** is optimal.
  - Under positive selection, a **subsidy** is optimal.

Application to Vaccination

Unknown/negative selection. How to choose between a mandate and no intervention?

$$\underbrace{\mu}_{\text{benefit guaranteed by a mandate}} \gtreqless \underbrace{\mathbf{E}_F[[c - v(\theta)]_+]}_{\text{allocative loss from a mandate}} .$$

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HPV/mpox: Individuals anticipating greater exposure may have higher willingness to pay.

Extension #2: Revenue Concerns

The regulator maximizes worst-case external benefit plus $\lambda \, \mathbf{E}_F[t(\theta) - cq(\theta)]$, subject to

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This constraint limits the burden on agents as a group: **aggregate direct utility** $\geq \underline{U}$.

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- Under unknown or negative selection, a **quantity floor** is uniquely optimal.
- Under positive selection, a **uniform subsidy** of μ/λ is uniquely optimal.

Extension #3: Bounded Marginal Externalities

Suppose $\xi \in [\underline{\xi}, \overline{\xi}]$ is known, where $0 \leq \underline{\xi} \leq \mu \leq \overline{\xi}$.

Nature can no longer concentrate all marginal benefit on an arbitrarily small set of agents.

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- Above $\underline{q}$: consume $D(c - \underline{\xi}, \theta)$ (these are agents with strong consumption preferences).
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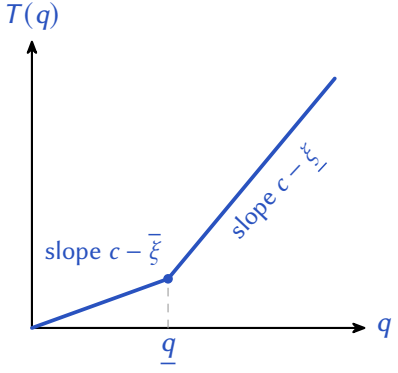
The optimal allocation is implemented by a **two-tier subsidy**:

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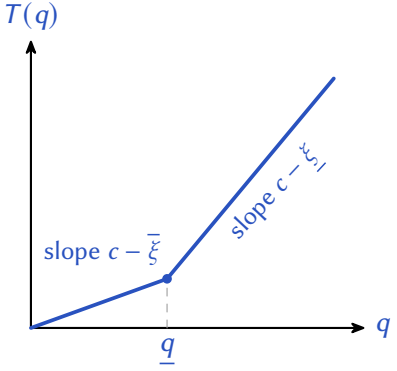
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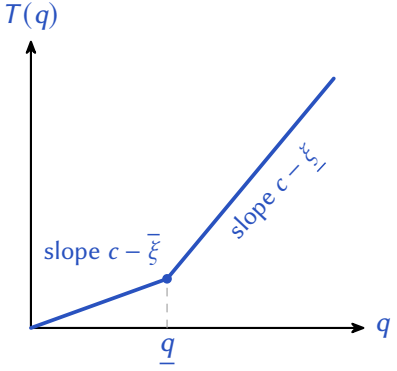
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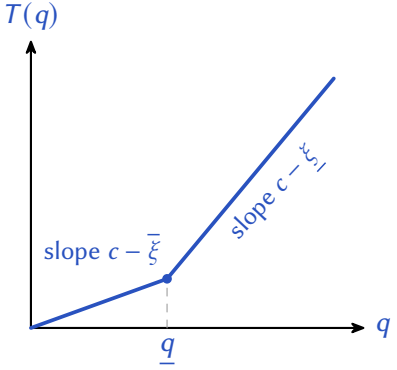
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Other Applications

As the Supplemental Appendix shows, the logic extends beyond consumption externalities.

- **Regulatory uncertainty** *à la* Weitzman (1974).

  Regulator knows average marginal benefit, but not its dependence on abatement costs.

  Even with nonlinear payment schedules, a **quantity control** is robustly optimal.

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  Even with nonlinear payment schedules, a **quantity control** is robustly optimal.

- **Costly screening** (e.g., for redistribution).

  Regulator knows MRS distribution and average waiting cost, but not joint distribution.

  Robustness can favor **randomization**: a uniform lottery with no costly screening.

  This complements a growing literature on Bayesian redistributive mechanism design.

Conclusion

Much of the prices vs. quantities debate considers **simple instruments**, **rich information**.

As a subsequent literature shows, richer price instruments can dominate quantity controls.

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This paper differs along both dimensions: **rich instruments**, **simple information**.

- Heterogeneity + robustness provide a welfare-based rationale for **quantity controls**.
- Information about the direction of selection can justify a **uniform tax** or **subsidy**.

$\rightsquigarrow$ Instrument choice (prices vs. quantities) turns on what is known about **selection**.

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