Pareto-Improving Pricing:
Why 3 Is Better Than 2

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APIOC

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Motivation

Waiting (due to congestion) is common in resource allocation:

- Internet traffic;
- vehicular traffic;
- security lines at airports; and
- wait lists (e.g., public housing, healthcare, DMV).

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Equity-efficiency tradeoff associated with **pricing priority**:

- gains from allocating priority to the most time-pressed agents…
- …but losses from making things worse for poorer agents.

WEALTH CONCENTRATION

HOT lanes make my blood boil: Fast tracks for the rich on the rise

ARTICLES

OCTOBER 31, 2012

by Salvatore Babones

Motivation

The Lexus Lanes — and why they won't work

Instead of rewarding carpools and getting people out of private cars, we are rewarding wealth and encouraging more people to drive. How does this make sense?

By
Zelda Bronstein
-
October 19, 2017

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Motivation

Why Some New York City Residents Are Suing Over Congestion Pricing

Their lawsuits argue that the tolling program would shift traffic and pollution to poor and minority neighborhoods and hurt small businesses.

Listen to this article · 6:44 min Learn more

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Congestion pricing in Manhattan is scheduled to start on June 30, but a slate of lawsuits seek to thwart those plans. Andrea Mohin/The New York Times

By Winnie Hu

Published May 16, 2024 Updated May 19, 2024

MONEY

Why must we pay to have a slightly less miserable time at the airport?

TSA PreCheck, Clear, and how the airport splits travelers into haves and have-nots.

by **Emily Stewart**
May 19, 2022, 7:00 AM CDT

Travelers wait in a long queue at the security checkpoint of Orlando International Airport the day before the Thanksgiving holiday on November 24, 2021. Paul Hennessy/SOPA Images/LightRocket via Getty Images

Emily Stewart covered business and economics for Vox and wrote the newsletter The Big Squeeze, examining the ways ordinary people are being squeezed under capitalism. Before joining Vox, she worked for TheStreet.

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$\rightsquigarrow$ 1 fast lane + 1 slow lane are not enough; some agents will be worse off.

#2. If this tradeoff is not too severe, **a Pareto improvement with 3 priority tiers** exists.

(Seed of a) Literature Review

#1. Possibility of Pareto improvements in priority service

Chao and Wilson (1987), Gershkov and Schweinzer (2010), Hall (2018), Gershkov and Winter (2022), …

#2. Market-design approach to congestion pricing

Ostrovsky and Schwarz (2018), Ostrovsky and Yang (2024), …

#3. Inequality-aware market design

Weitzman (1977), Condorelli (2013), Dworczak ⓡ Kominers ⓡ Akbarpour (2021, 2024), Kang (2023), Kang and Watt (2024), …

#4. Recent empirical papers on congestion

Hanna, Kreindler, Olken (2017), Kreindler (2024), Cook and Li (2024), …

Setup

Model

There is a unit mass of agents with private **marginal rate of substitution** (MRS) $r \in [\underline{r}, \overline{r}]$ between wait time and money.

- $\rightsquigarrow$ MRS r has a CDF G with strictly positive density g on $[\underline{r}, \overline{r}]$.

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$\rightsquigarrow$ MRS r has a CDF G with strictly positive density g on $[\underline{r}, \bar{r}]$.

An agent with MRS r who obtains a service/good after waiting t and paying p gets **utility**

$$V - r \cdot t - p,$$

where V is some known (sufficiently large) constant.

Think of t as measuring the **cost** of waiting; we can accommodate the case when $t = c(\tau)$, where c is an increasing function and τ is physical wait time.

Model

Designer chooses a distribution of wait times F from a feasible set of distributions $\mathcal{F}$.

Any choice of $F \in \mathcal{F}$ pins down the outcomes of a budget-balanced, incentive-compatible mechanism with transfers:

- Allocation rule is given by $t(r) = F^{-1}(1 - G(r))$.
- Payments are pinned down by the envelope formula:

$$U(r) \equiv V - r \cdot t(r) - p(r) = \overline{U} - \int_{\underline{r}}^{r} t(s)\, \mathrm{d}s.$$

- Constant $\overline{U}$ pinned down by budget balance.

Running Example

There are N lanes on a highway.

If mass m_k of agents drive in lane k, wait time in lane k is $w(m_k)$, where $w' > 0$ and $w'' > 0$.

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Let F_m be the distribution of wait times induced by m (ordered so that $m_i \leq m_{i+1}$).

$\rightsquigarrow$ F_m assigns mass m_1 to $w(m_1)$, mass m_2 to $w(m_2)$, etc.

Then, the set of feasible distributions is

$$\mathcal{F} = \left\{ F \in \Delta(\mathbb{R}_+) : F \in \mathrm{MPC}(F_m) \text{ for some } m \in \Delta^{N-1} \right\}.$$

Unpriced benchmark (laissez-faire allocation)

Suppose that we set

$$m = (1/N, \ldots, 1/N).$$

The corresponding F_m is a Dirac delta at $w(1/N)$.

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The unique mechanism corresponding to F_m:

#1. charges everyone a price of zero; and

#2. assigns wait time $w(1/N)$ to everyone.

Example of nontrivial mechanism

Suppose that $N = 3$, $r \sim \mathscr{U}[0, 1]$, and we set

$$m = (1/5,\ 2/5,\ 2/5).$$

The corresponding F_m has mass $1/5$ of wait time $w(1/5)$ and mass $4/5$ of wait time $w(2/5)$.

$\rightsquigarrow$ Types $r \in [4/5, 1]$ get wait time $w(1/5)$ and pay p_F.

$\rightsquigarrow$ Remaining types $r \in [0, 4/5]$ get wait time $w(2/5)$ and pay p_S.

Relative price pinned down by: $-\frac{4}{5} w\left(\frac{2}{5}\right) - p_S = -\frac{4}{5} w\left(\frac{1}{5}\right) - p_F$.

Budget balance: $\frac{1}{5} p_F + \frac{4}{5} p_S = 0$.

Assumptions on $\mathcal{F}$

Axiom ("Randomization").

The set $\mathcal{F}$ is closed under mean-preserving contractions:

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- $\rightsquigarrow$ We can always assign wait times randomly.
- $\rightsquigarrow$ True in the running example by design (as long as we can "randomize" access to lanes).

Axiom ("Friction").

There exists t_0 such that $\delta_{t_0} \in \mathcal{F}$ (the **unpriced benchmark**) **uniquely minimizes** the average wait time $\mathbf{E}_F[t]$ across all feasible $F \in \mathcal{F}$.

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- $\rightsquigarrow$ Any mean-preserving spread of δ_{t_0} must take us outside of $\mathcal{F}$.
- $\rightsquigarrow$ There is a **tradeoff between dispersion and average wait time**.
- $\rightsquigarrow$ True in the running example if $w(\cdot)$ is strictly convex (around $1/N$).

Assumptions on $\mathcal{F}$

Axiom ("Smoothness").

For every sufficiently small $\varepsilon > 0$, there exist $F \in \mathcal{F}$ and $q \in (0, 1)$ such that

$$F^{-1}(q + \sqrt{\varepsilon}) < t_0 \quad \text{and} \quad \frac{\mathbf{E}_F[t] - t_0}{q\,(t_0 - \mathbf{E}_F[t \mid F(t) \leq q])} < \varepsilon.$$

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$\leadsto$ Tradeoff between dispersion and average wait time is not too severe.

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⇝ Tradeoff between dispersion and average wait time is not too severe.

⇝ True in the running example if $w(\cdot)$ is continuously differentiable (around $1/N$).

Construction: $(1/N) - \delta$ agents use lane 1, $(1/N) + \delta$ agents use lane N, take $\delta \to 0$.

Assumptions on $\mathcal{F}$

We will sometimes refer to the opposite assumption for contrast:

Axiom ("No Friction").

The set $\mathcal{F}$ is closed under mean-preserving spreads:

$$F \in \mathcal{F} \implies \mathsf{MPS}(F) \subseteq \mathcal{F}.$$

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Axiom ("No Friction").

The set $\mathcal{F}$ is closed under mean-preserving spreads:

$$F \in \mathcal{F} \implies \mathsf{MPS}(F) \subseteq \mathcal{F}.$$

$\rightsquigarrow$ "No Friction" implies a stronger version of "Smoothness":

There exists $q > 0$ such that, for every $\varepsilon > 0$ small, there exist $F \in \mathcal{F}$ satisfying

$$F^{-1}(q) < t_0 \quad \text{and} \quad \frac{\mathbf{E}_F[t] - t_0}{t_0 - \mathbf{E}_F[t \mid F(t) \leq q]} < \varepsilon.$$

This is because we can find F such that $\mathbf{E}_F[t] = t_0$.

Main Result

We can **Pareto-improve** upon a mechanism delivering utilities $U(r)$ if there is a feasible mechanism delivering utilities $\widetilde{U}(r)$ such that $\widetilde{U}(r) \geq U(r)$, for all r, and strictly for some r.

Preliminaries

We can **Pareto-improve** upon a mechanism delivering utilities $U(r)$ if there is a feasible mechanism delivering utilities $\widetilde{U}(r)$ such that $\widetilde{U}(r) \geq U(r)$, for all r, and strictly for some r.

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Despite quasilinearity, Pareto optimality is **not** equivalent to maximizing sum of utilities.

- $\rightsquigarrow$ Inequality-aware market design: mechanisms on the (IC-constrained) Pareto frontier may involve **random allocation**.

Definition.

We say that $F \in \mathcal{F}$ offers n (priority) tiers if $|\text{supp}(F)| = n$.

Results

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Theorem.

Assume Randomization, Friction, and Smoothness.

There exists a strict Pareto improvement over the unpriced benchmark that uses 3 tiers.

It is not possible to (weakly) Pareto-improve upon the unpriced benchmark with 2 tiers.

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The first result uses only "Randomization" + "Smoothness"; the second uses only "Friction."

Discussion of Results

In applications, we often see priority systems with 2 tiers:

- fast lane + slow lane;
- single fast-track option (e.g., Clear).

Our theorem implies that these priority systems must make some agents worse off.

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Our theorem implies that these priority systems must make some agents worse off.

“Friction” results in a tradeoff between average wait time and dispersion.

Intuitively, 2 tiers are insufficient to resolve this tradeoff while improving for all agents.

Discussion of Results in Running Example

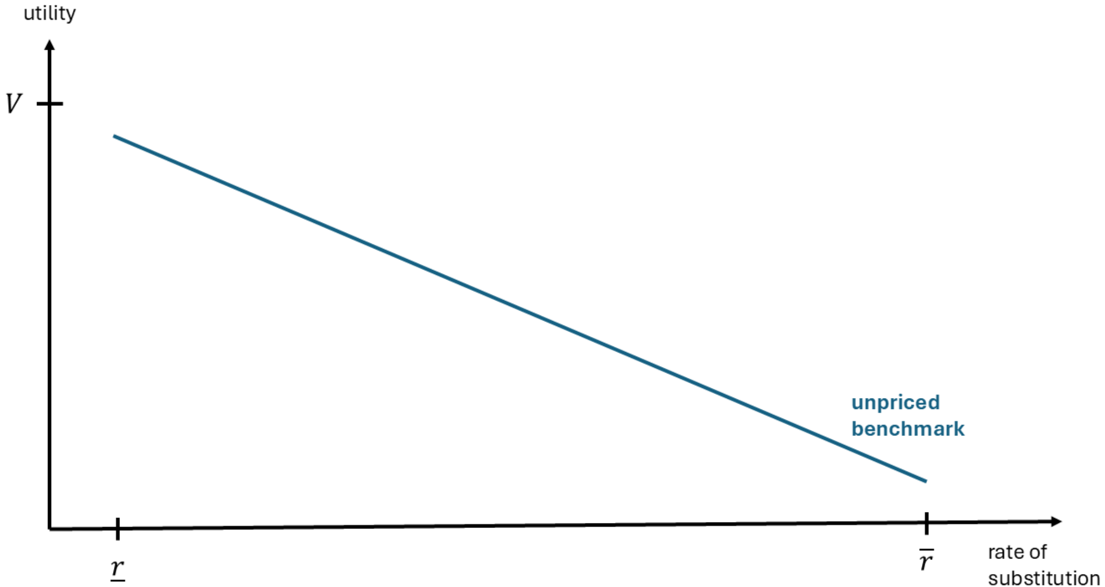
In the running example:

- if there are 3 or more lanes, there exists a deterministic Pareto-improving mechanism;
- if there are only 2 lanes, no deterministic mechanism can be a Pareto improvement.

However, even with 2 lanes, Pareto improvement is possible with a stochastic mechanism (that offers 3 options).

Proof of Main Result

Proof



utility
V
unpriced
benchmark
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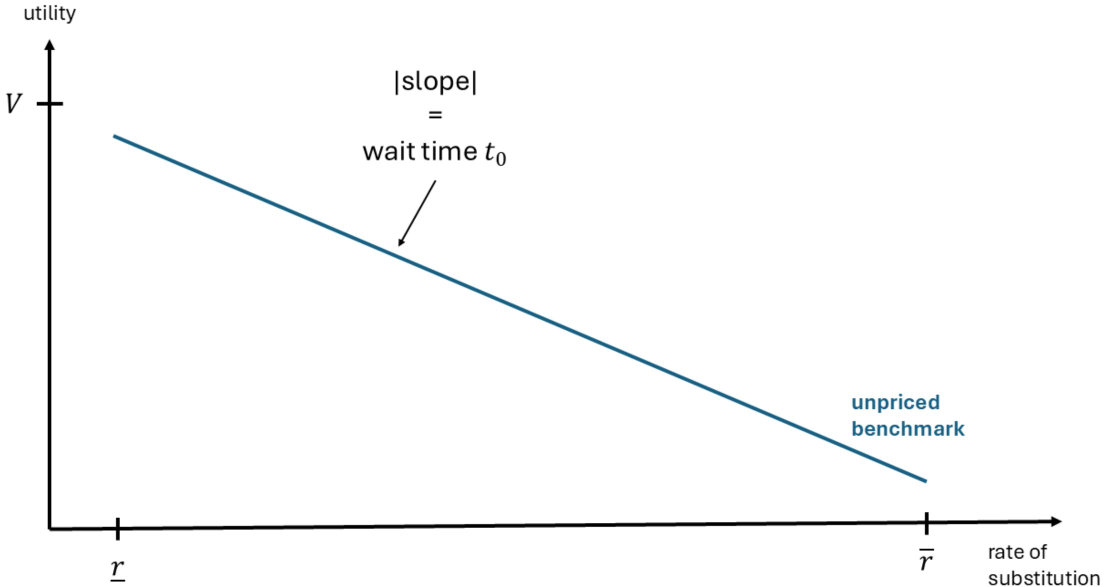
$\overline{r}$
rate of
substitution

Proof



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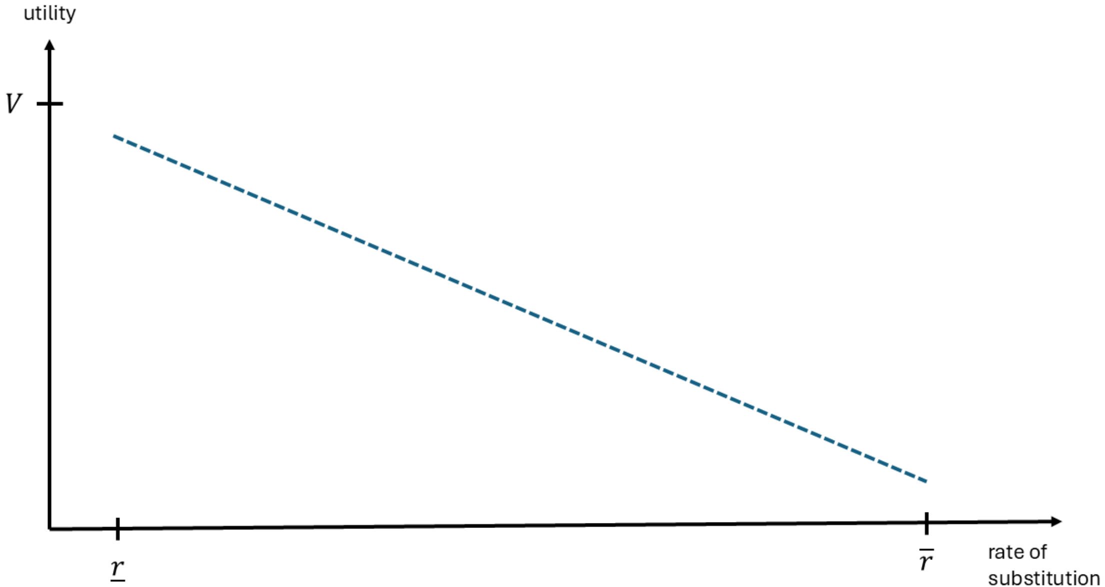
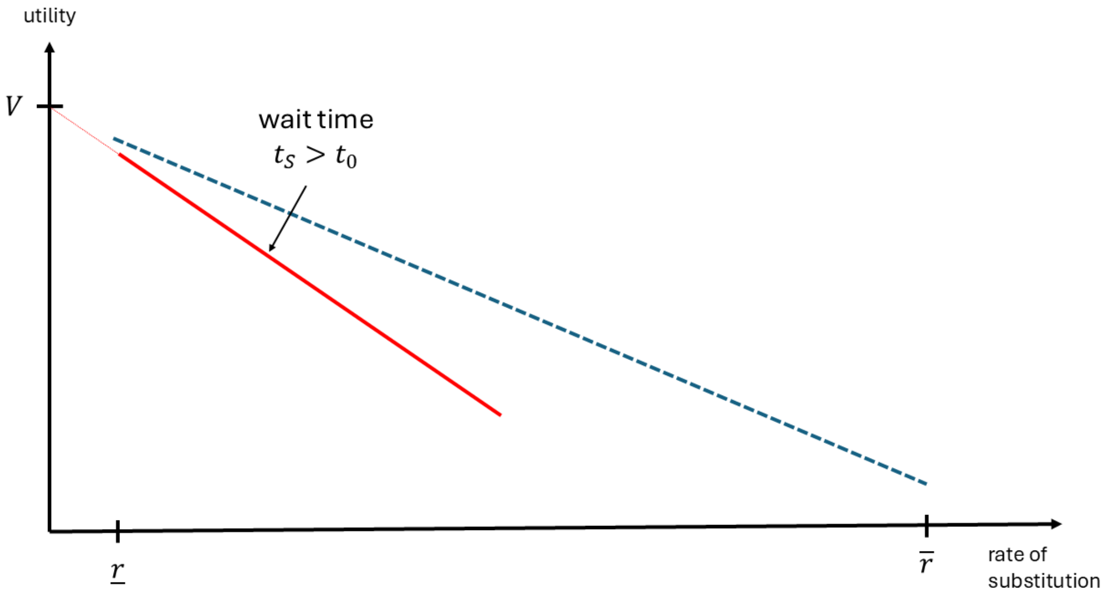
Proof



Proof



utility
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wait time
$t_S > t_0$
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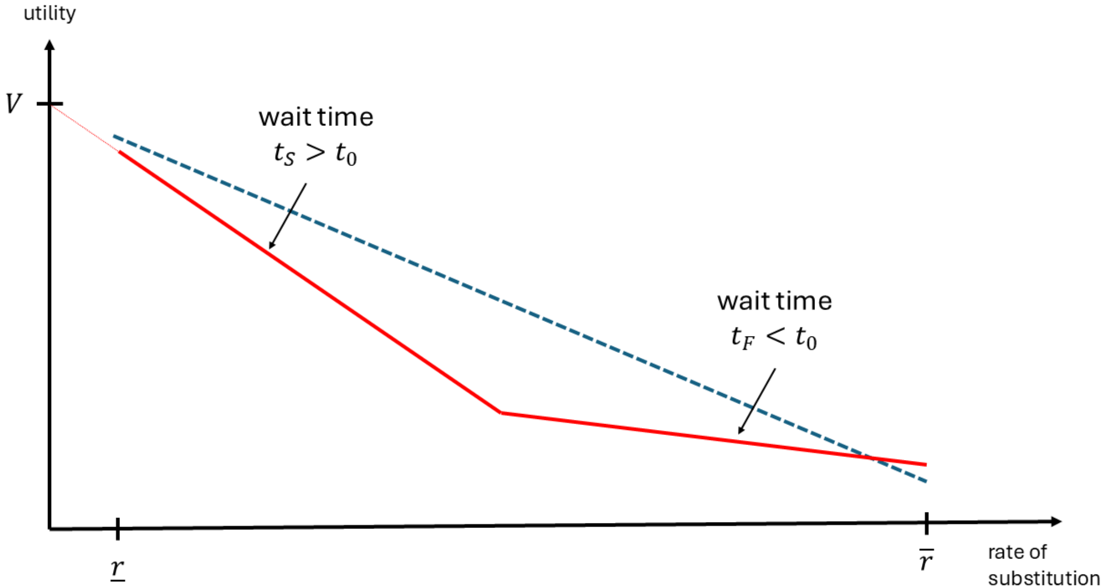
rate of
substitution

Proof



utility
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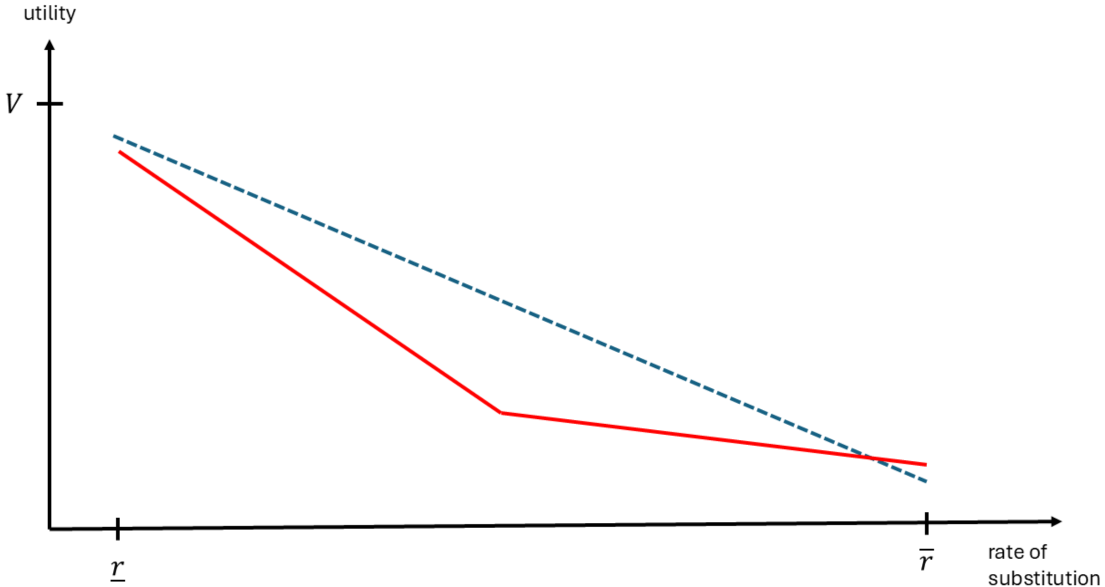
wait time
$t_S > t_0$
wait time
$t_F < t_0$
$\underline{r}$
$\overline{r}$
rate of
substitution

Proof



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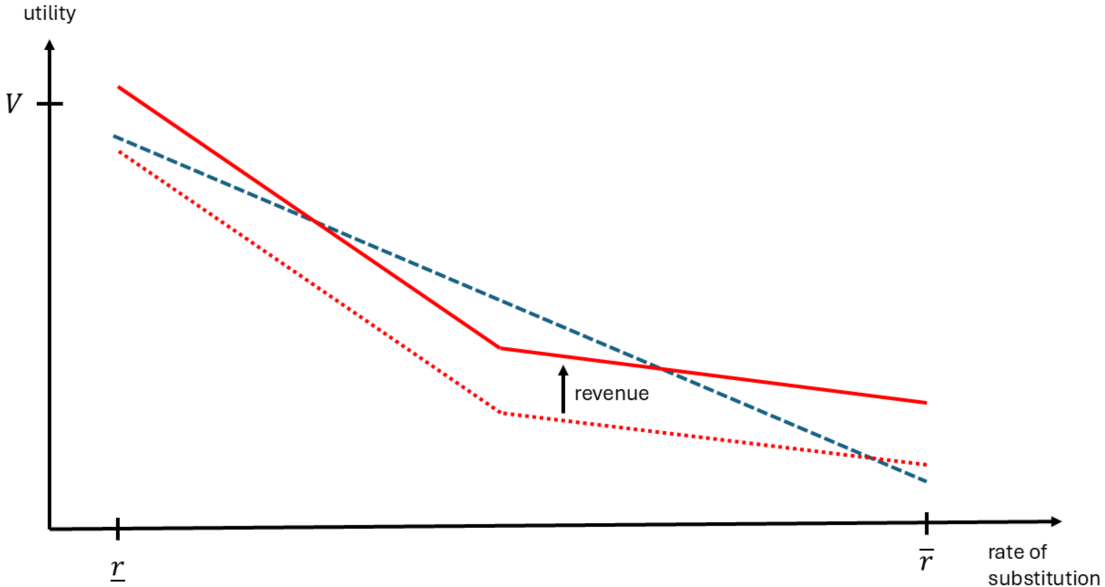

utility
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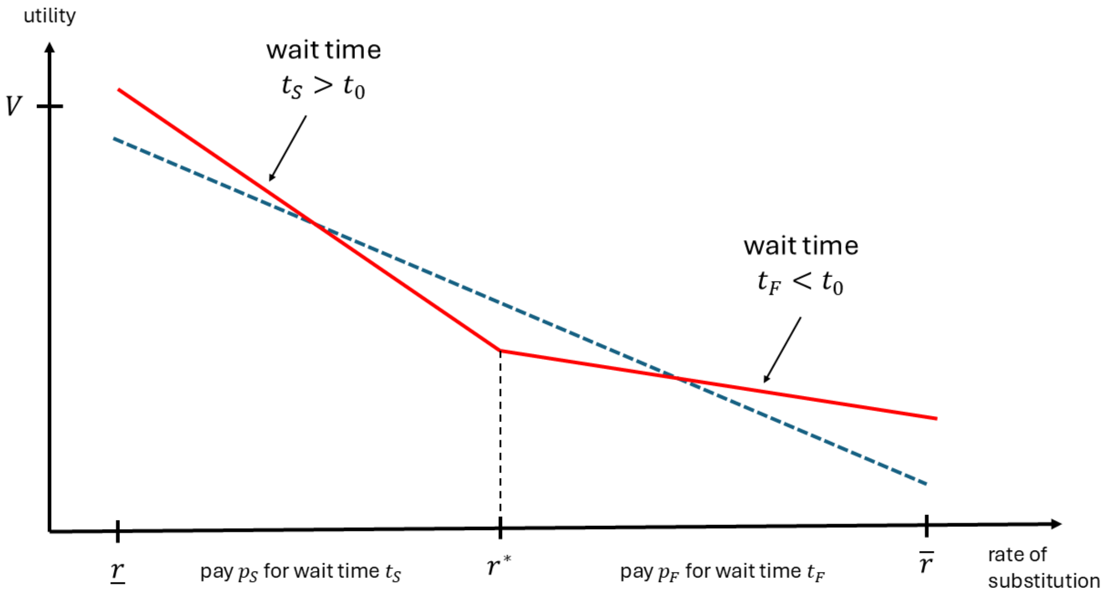
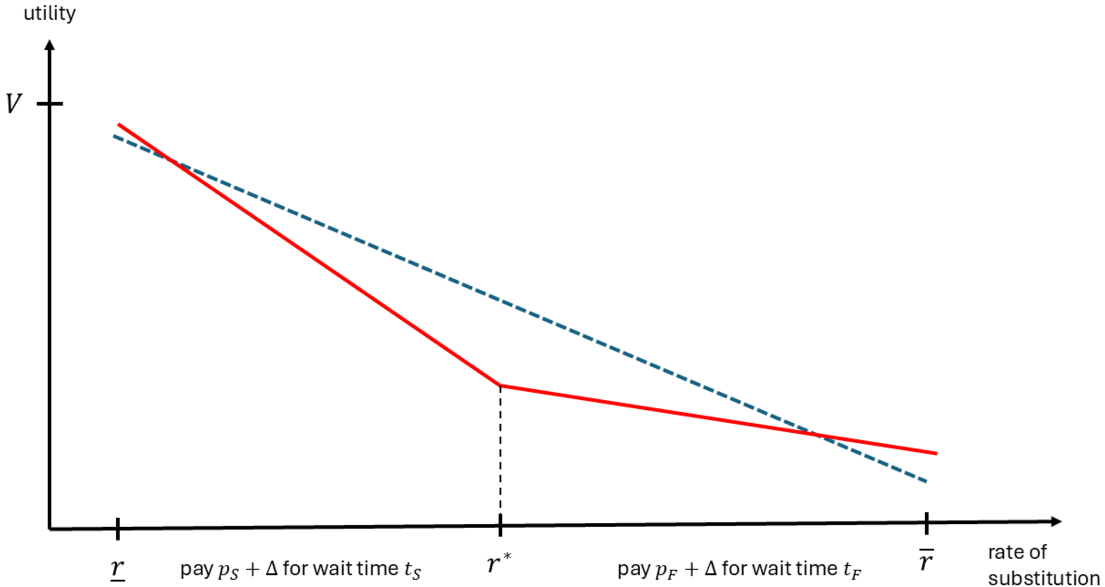
$\underline{r}$
$\overline{r}$
rate of substitution

Proof

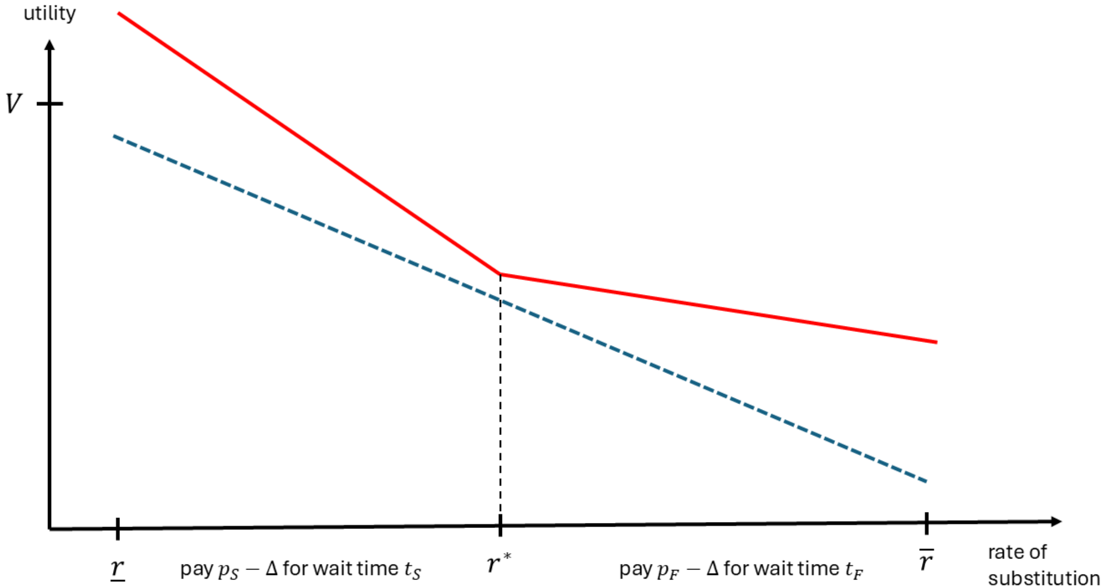

utility
V
wait time
$t_S > t_0$
wait time
$t_F < t_0$
$\underline{r}$
pay p_S for wait time t_S
r^*
pay p_F for wait time t_F
$\overline{r}$
rate of
substitution

Proof



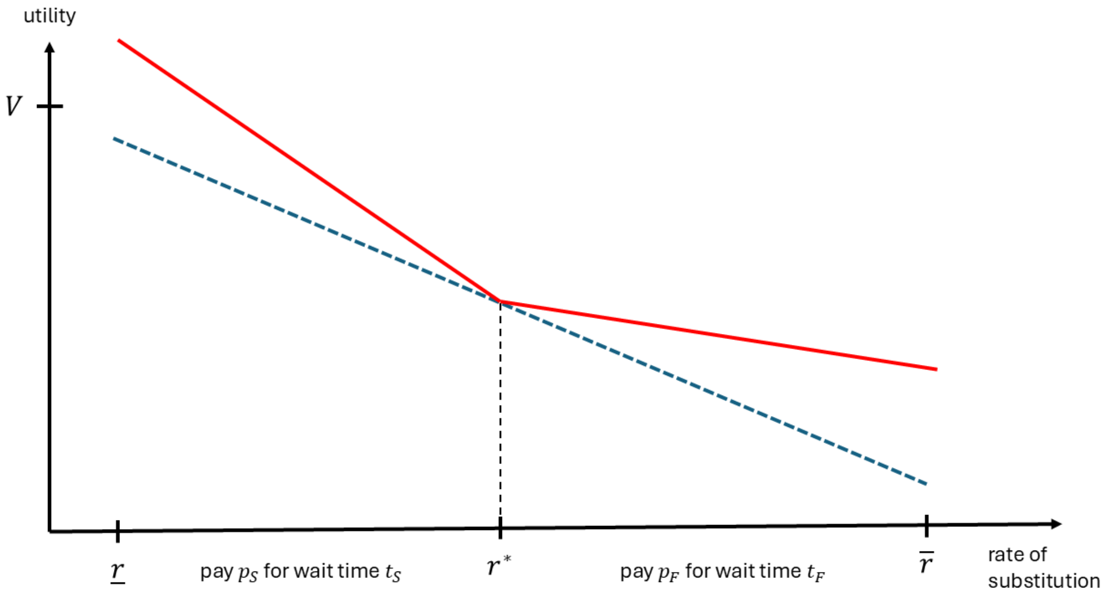
utility
V
$\underline{r}$
pay $p_S + \Delta$ for wait time t_S
r^*
pay $p_F + \Delta$ for wait time t_F
$\overline{r}$
rate of
substitution

Proof



utility
V
$\underline{r}$
pay $p_S - \Delta$ for wait time t_S
r^*
pay $p_F - \Delta$ for wait time t_F
$\overline{r}$
rate of substitution

Proof



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r^*
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$\overline{r}$
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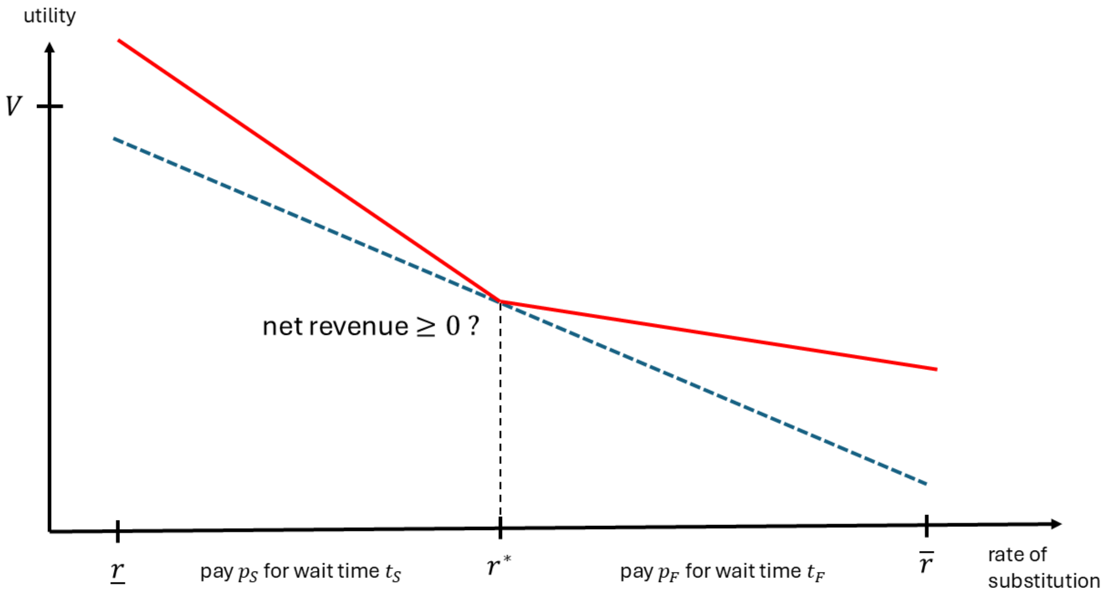
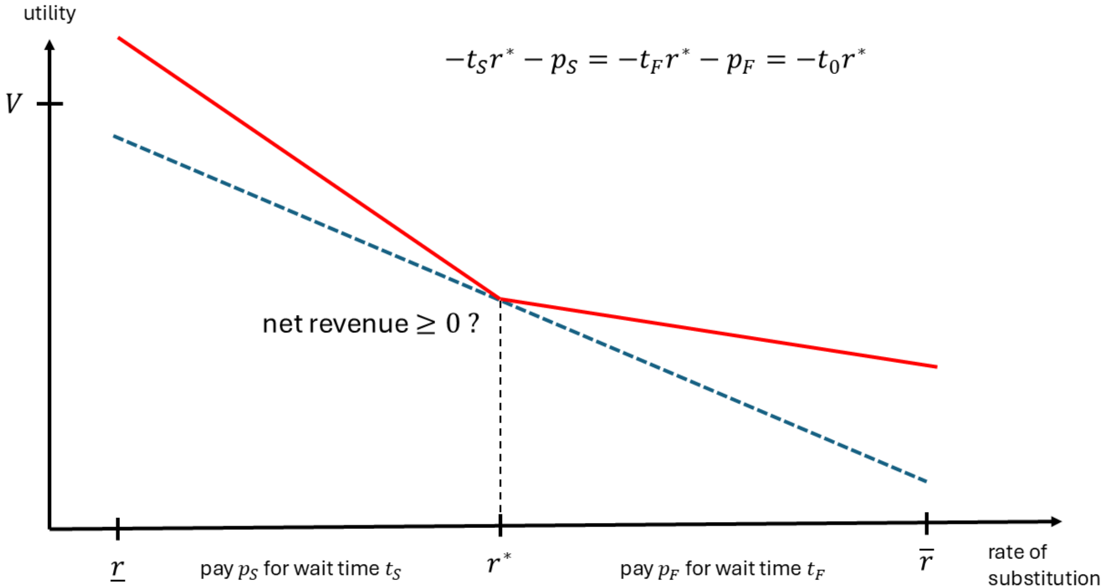


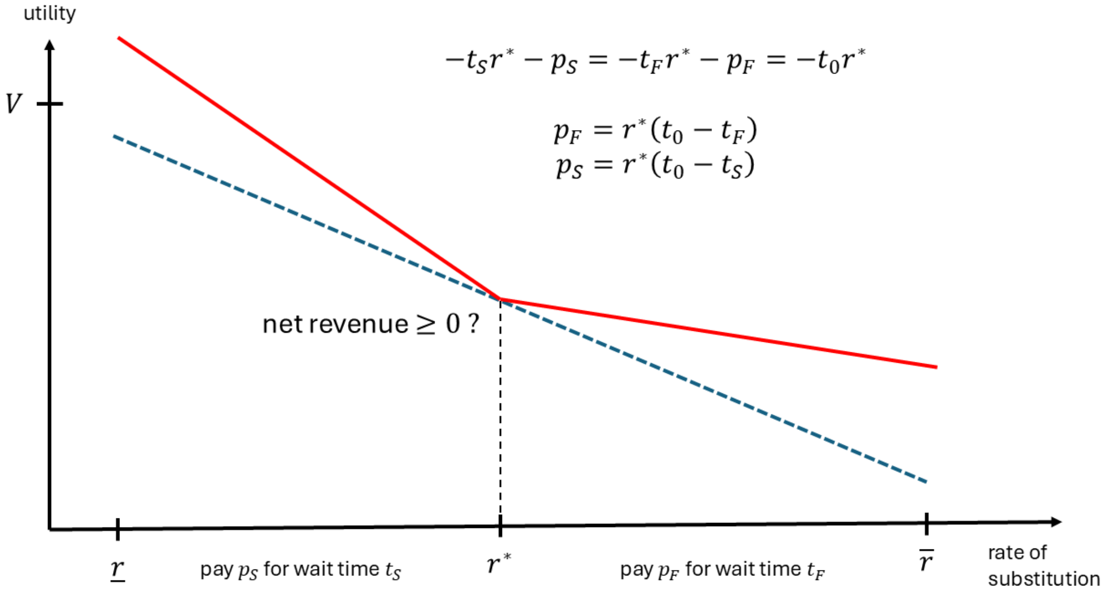
utility
V
net revenue ≥ 0 ?
$\underline{r}$
pay p_S for wait time t_S
r^*
pay p_F for wait time t_F
$\bar{r}$
rate of substitution

Proof



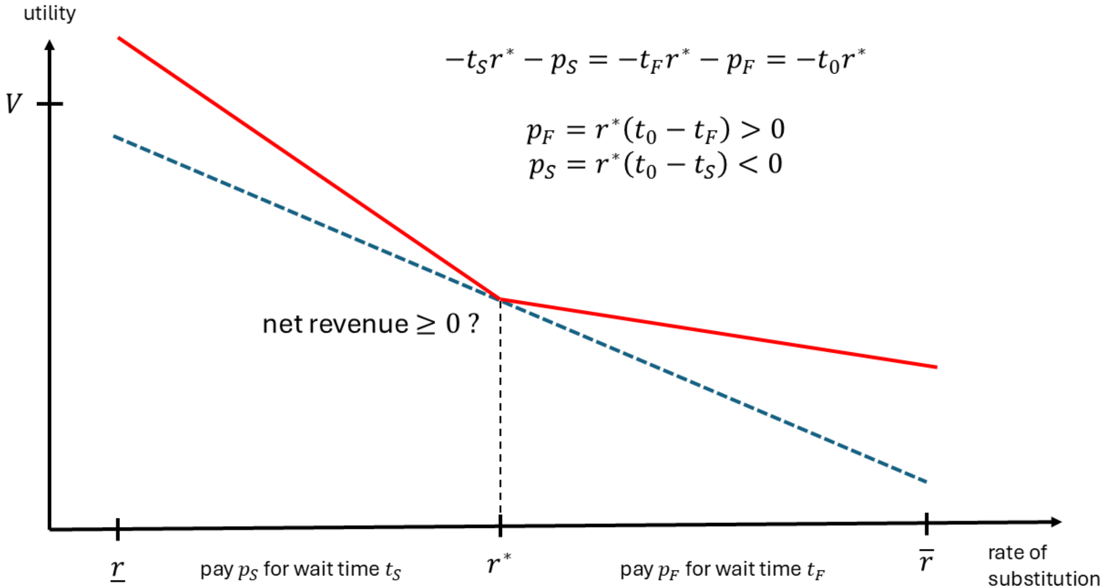
utility
$-t_S r^* - p_S = -t_F r^* - p_F = -t_0 r^*$
V
net revenue ≥ 0 ?
$\underline{r}$
pay p_S for wait time t_S
r^*
pay p_F for wait time t_F
$\bar{r}$
rate of substitution

Proof



utility
V
$-t_S r^* - p_S = -t_F r^* - p_F = -t_0 r^*$
$p_F = r^*(t_0 - t_F)$
$p_S = r^*(t_0 - t_S)$
net revenue ≥ 0 ?
$\underline{r}$
pay p_S for wait time t_S
r^*
pay p_F for wait time t_F
$\bar{r}$
rate of substitution

Proof



utility
V
$-t_S r^* - p_S = -t_F r^* - p_F = -t_0 r^*$
$p_F = r^*(t_0 - t_F) > 0$
$p_S = r^*(t_0 - t_S) < 0$
net revenue ≥ 0 ?
$\underline{r}$
pay p_S for wait time t_S
r^*
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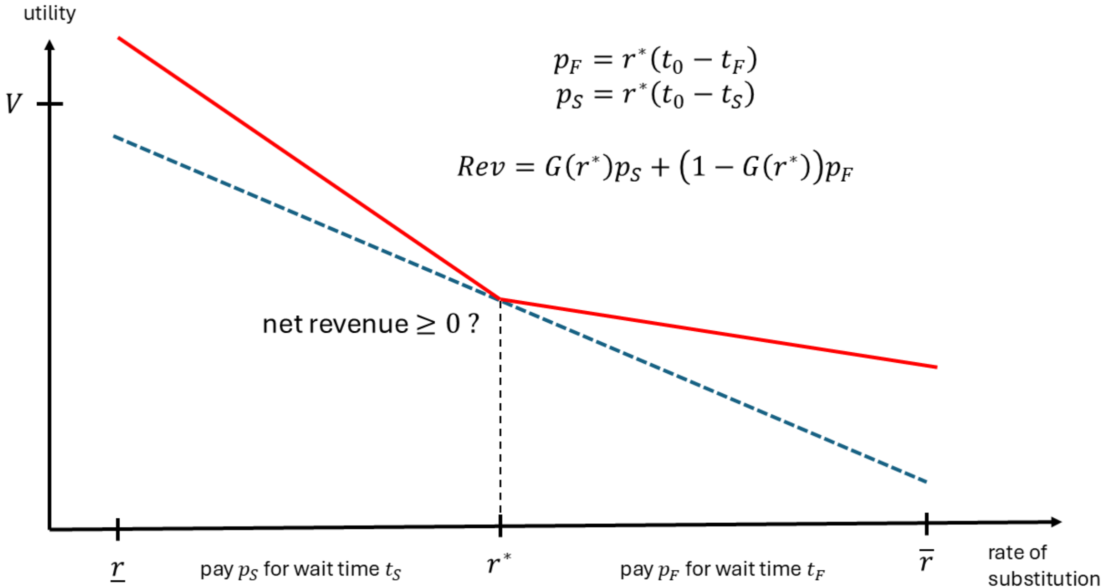
rate of substitution

Proof



utility
V
$p_F = r^*(t_0 - t_F)$
$p_S = r^*(t_0 - t_S)$
$Rev = G(r^*)p_S + \left(1 - G(r^*)\right)p_F$
net revenue ≥ 0 ?
$\underline{r}$
pay p_S for wait time t_S
r^*
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$\overline{r}$
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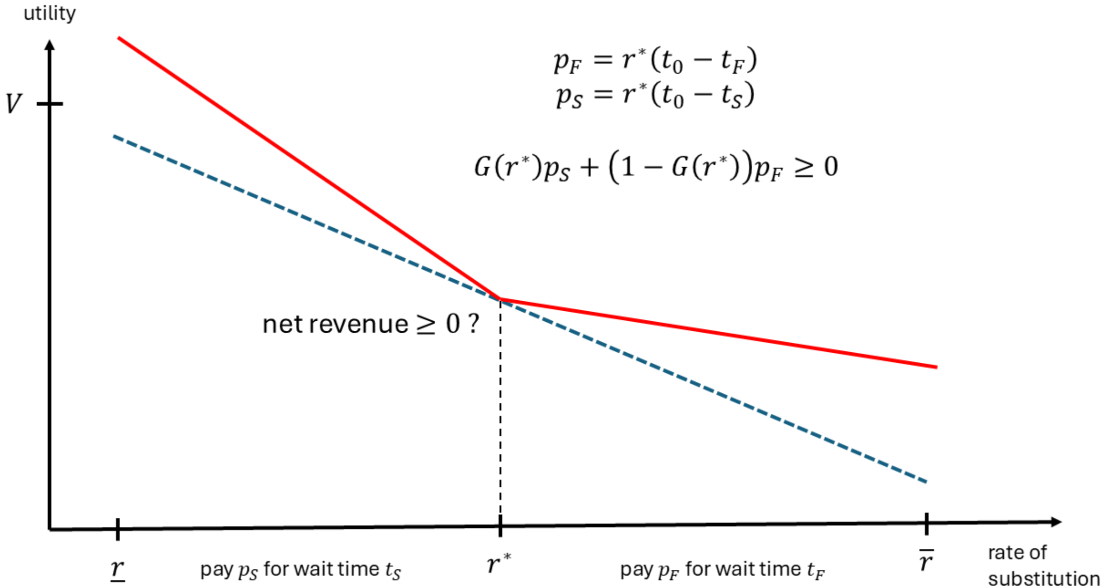
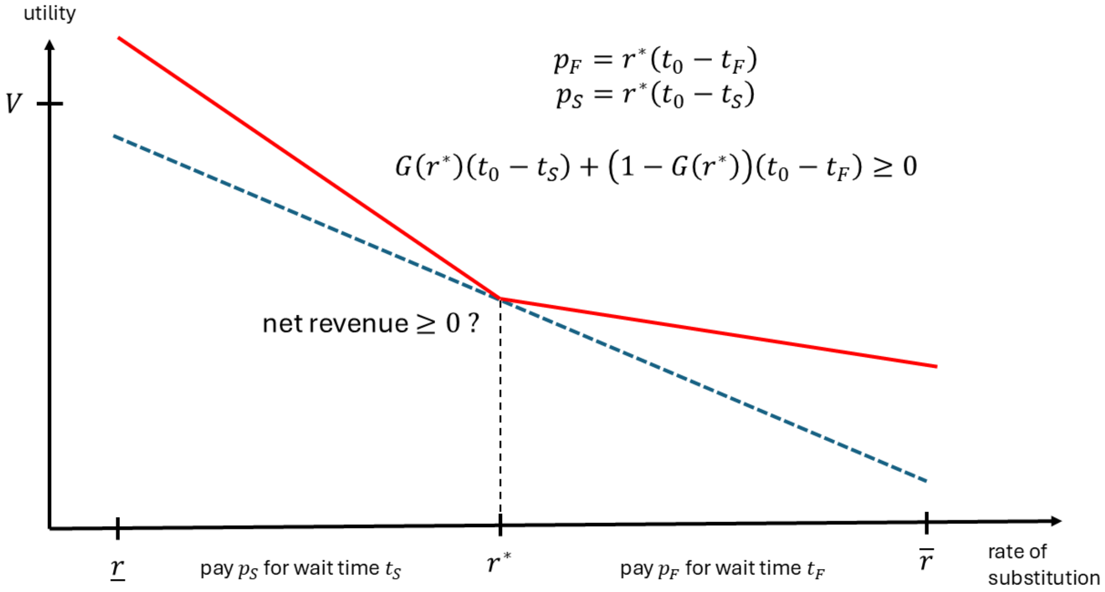


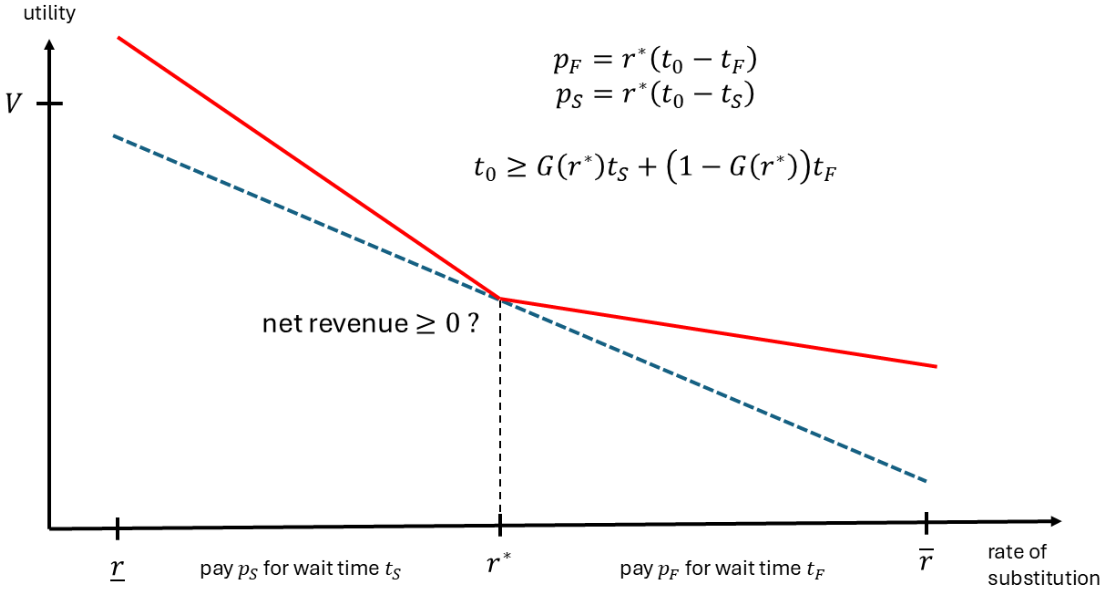
utility
V
$p_F = r^*(t_0 - t_F)$
$p_S = r^*(t_0 - t_S)$
$G(r^*)p_S + (1 - G(r^*))p_F \geq 0$
net revenue ≥ 0 ?
$\underline{r}$
pay p_S for wait time t_S
r^*
pay p_F for wait time t_F
$\bar{r}$
rate of substitution

Proof



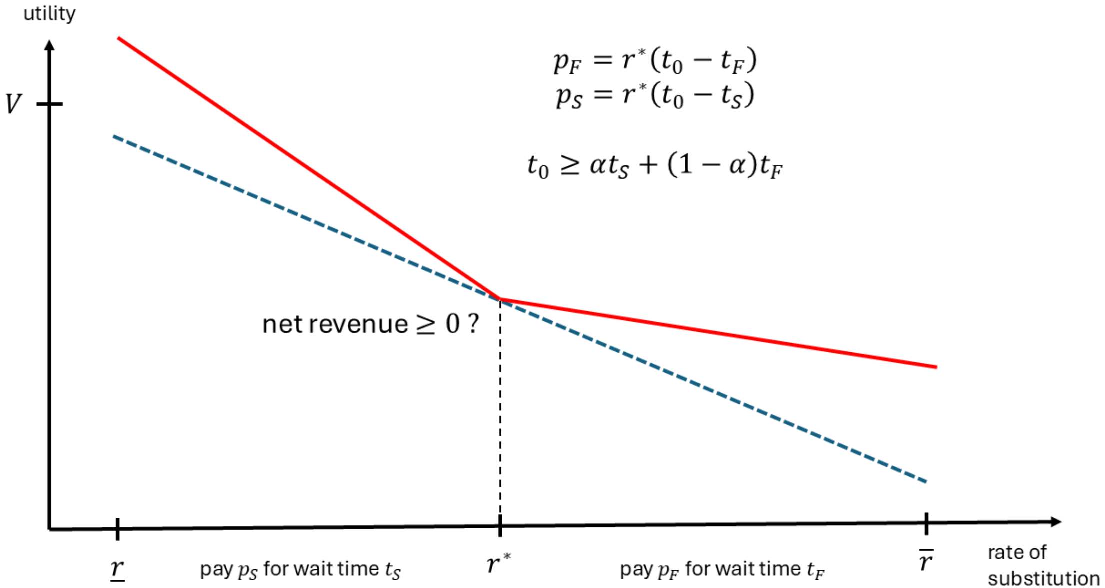
utility
V
$p_F = r^*(t_0 - t_F)$
$p_S = r^*(t_0 - t_S)$
$G(r^*)(t_0 - t_S) + \left(1 - G(r^*)\right)(t_0 - t_F) \geq 0$
net revenue ≥ 0 ?
$\underline{r}$
pay p_S for wait time t_S
r^*
pay p_F for wait time t_F
$\overline{r}$
rate of substitution

Proof

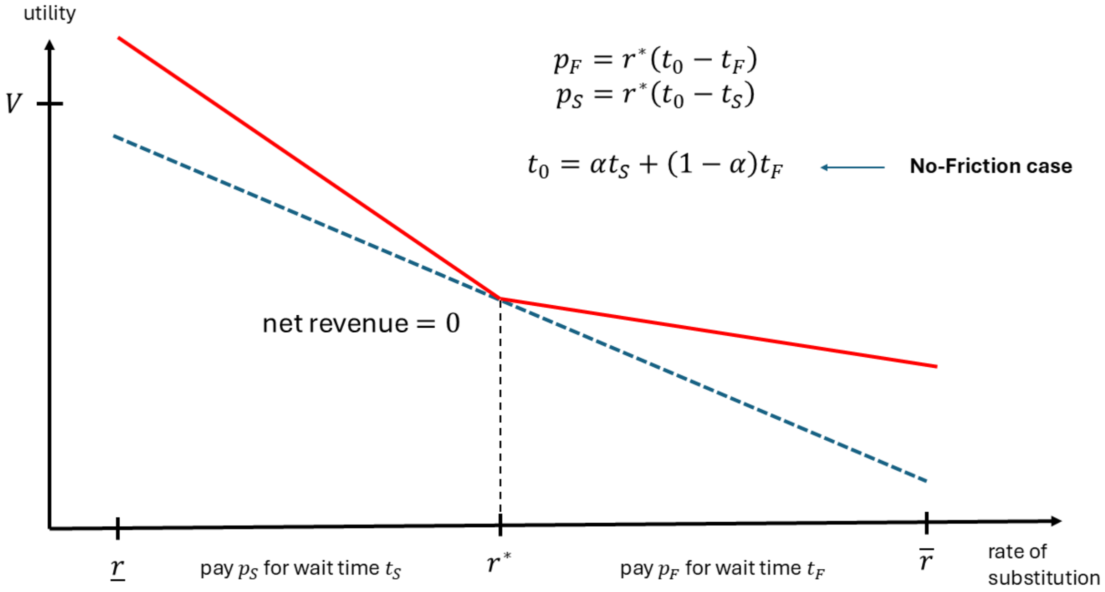


utility
V
$p_F = r^*(t_0 - t_F)$
$p_S = r^*(t_0 - t_S)$
$t_0 \geq G(r^*)t_S + \big(1 - G(r^*)\big)t_F$
net revenue ≥ 0 ?
$\underline{r}$
pay p_S for wait time t_S
r^*
pay p_F for wait time t_F
$\bar{r}$
rate of substitution

Proof

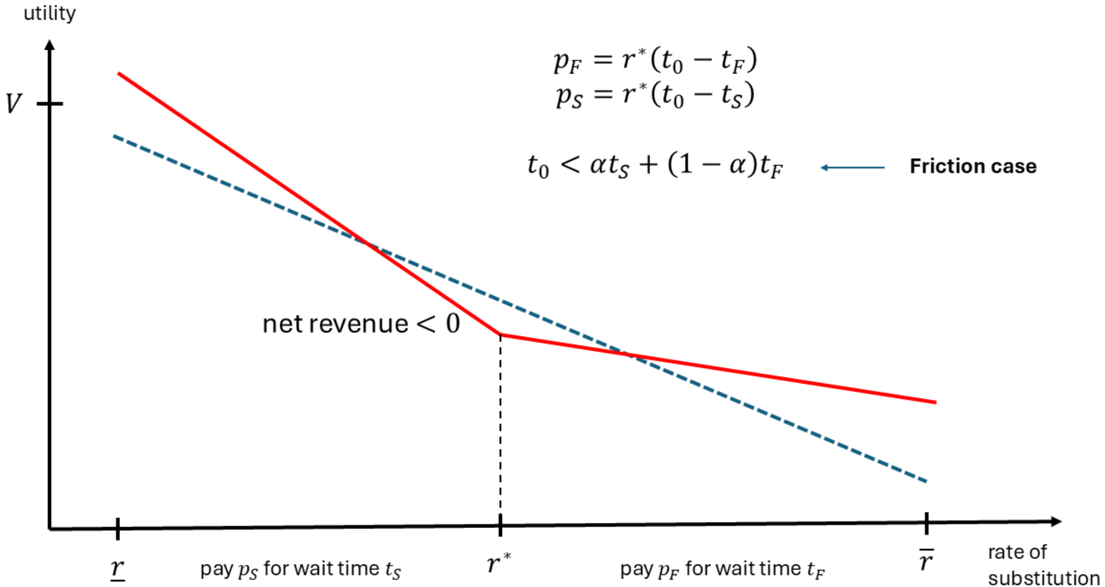

utility
V
$p_F = r^*(t_0 - t_F)$
$p_S = r^*(t_0 - t_S)$
$t_0 \geq \alpha t_S + (1-\alpha) t_F$
net revenue ≥ 0 ?
$\underline{r}$
pay p_S for wait time t_S
r^*
pay p_F for wait time t_F
$\overline{r}$
rate of substitution

Proof



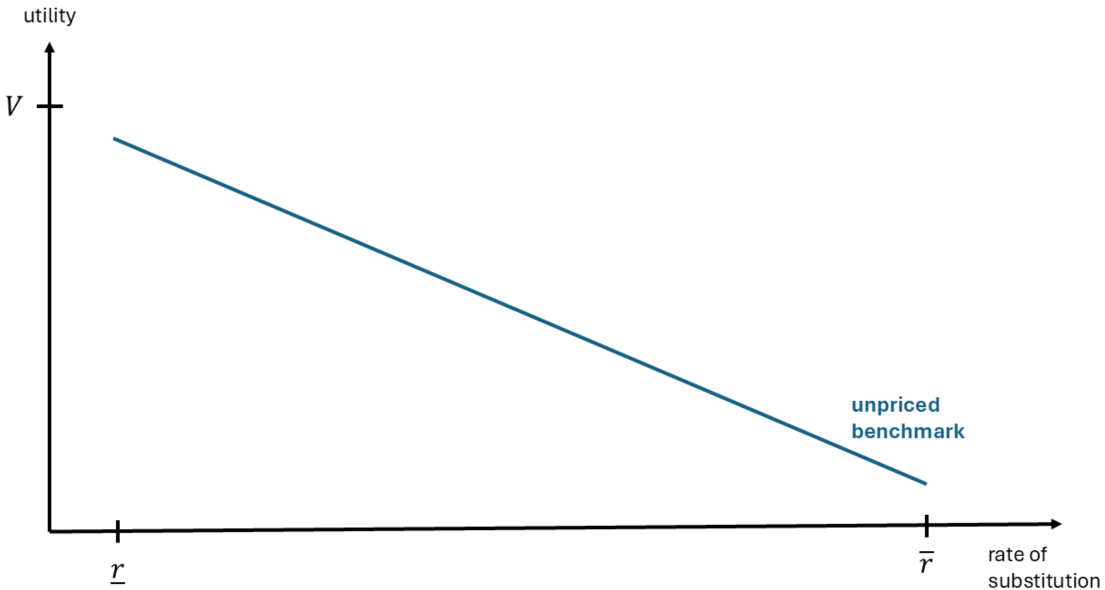
utility
V
$p_F = r^*(t_0 - t_F)$
$p_S = r^*(t_0 - t_S)$
$t_0 = \alpha t_S + (1-\alpha) t_F$
No-Friction case
net revenue = 0
$\underline{r}$
pay p_S for wait time t_S
r^*
pay p_F for wait time t_F
$\bar{r}$
rate of substitution

Proof



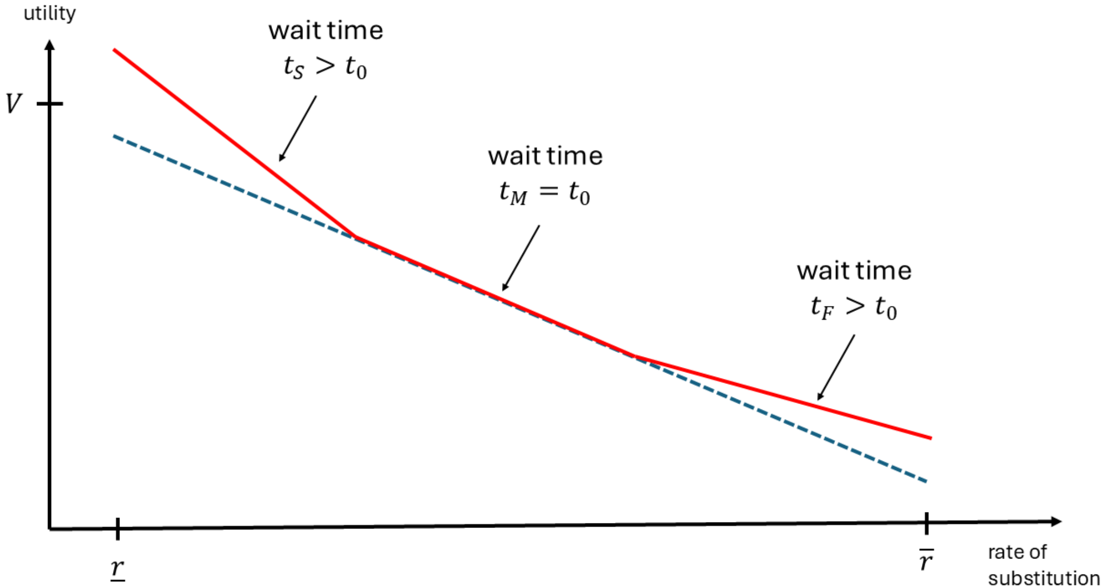
utility
V
$p_F = r^*(t_0 - t_F)$
$p_S = r^*(t_0 - t_S)$
$t_0 < \alpha t_S + (1-\alpha)t_F$
Friction case
net revenue < 0
$\underline{r}$
pay p_S for wait time t_S
r^*
pay p_F for wait time t_F
$\overline{r}$
rate of
substitution

Proof



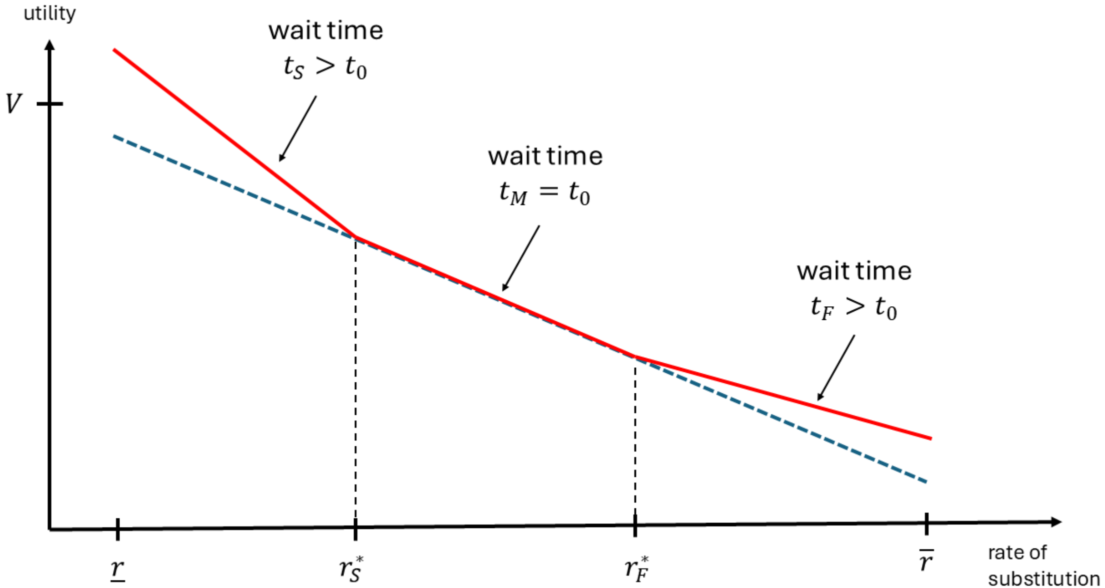
utility
V
unpriced
benchmark
$\underline{r}$
$\bar{r}$
rate of
substitution

Proof



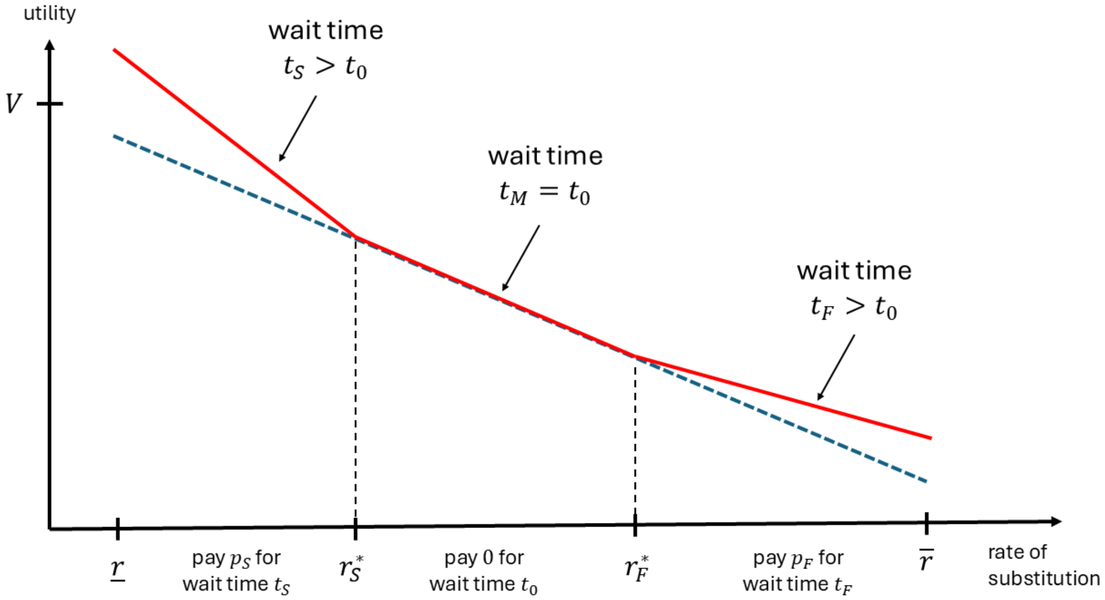
utility
V
wait time
$t_S > t_0$
wait time
$t_M = t_0$
wait time
$t_F > t_0$
$\underline{r}$
$\overline{r}$
rate of
substitution

Proof



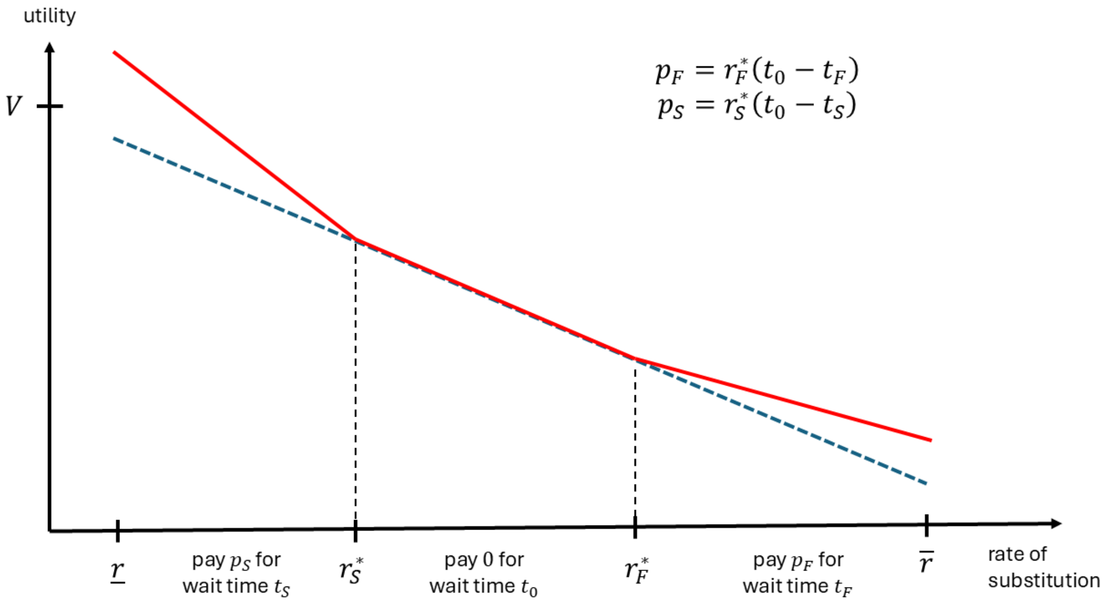
utility
V
wait time
$t_S > t_0$
wait time
$t_M = t_0$
wait time
$t_F > t_0$
$\underline{r}$
r_S^*
r_F^*
$\overline{r}$
rate of substitution

Proof



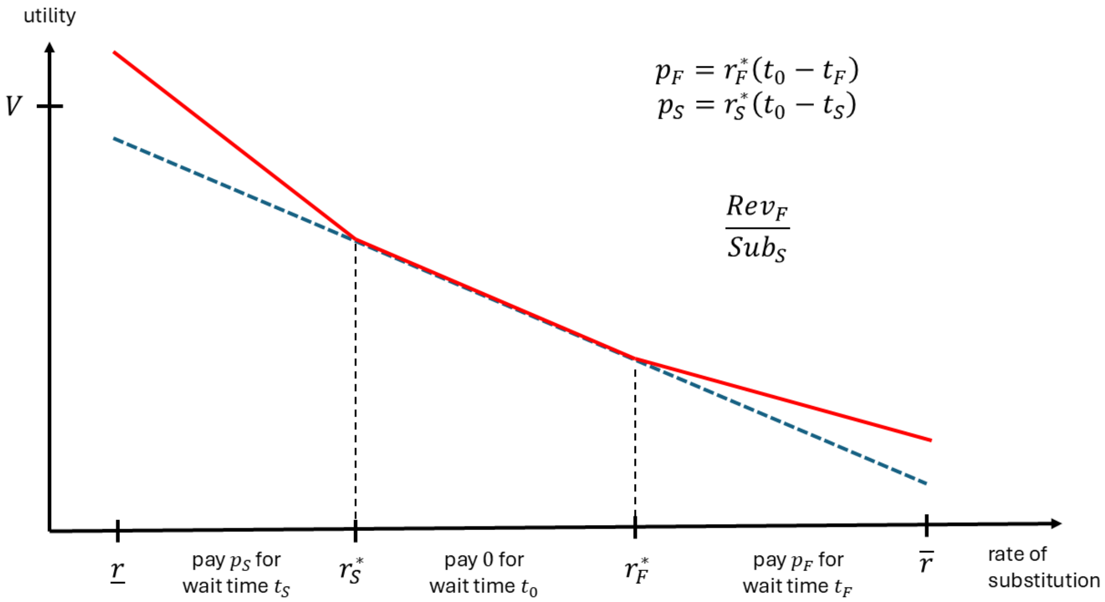
utility
V
wait time
$t_S > t_0$
wait time
$t_M = t_0$
wait time
$t_F > t_0$
$\underline{r}$
pay p_S for
wait time t_S
r_S^*
pay 0 for
wait time t_0
r_F^*
pay p_F for
wait time t_F
$\bar{r}$
rate of
substitution

Proof

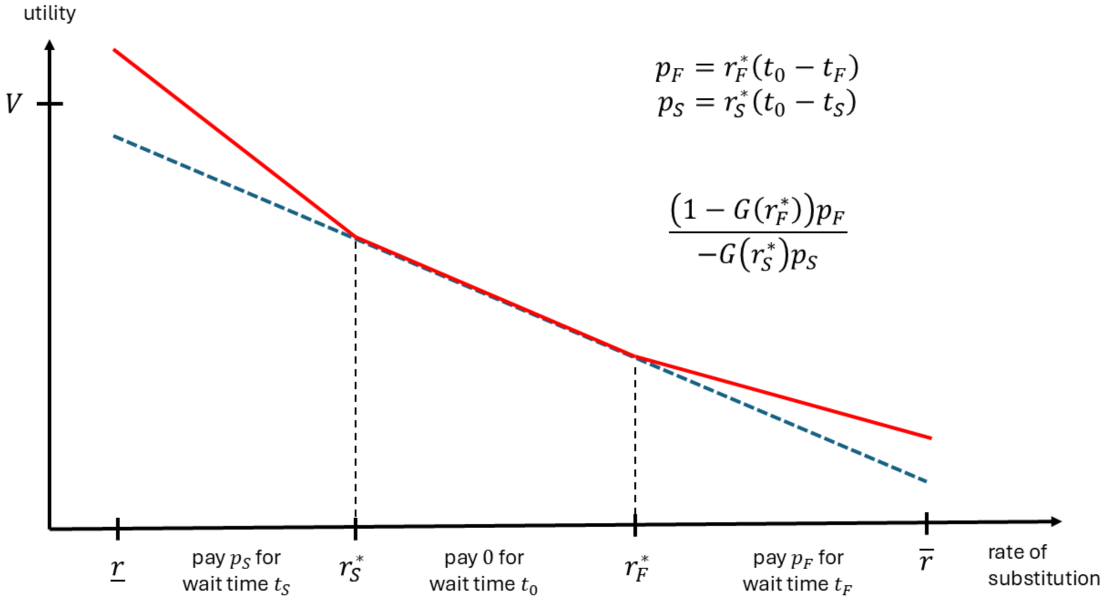


utility
V
$p_F = r_F^*(t_0 - t_F)$
$p_S = r_S^*(t_0 - t_S)$
$\underline{r}$
pay p_S for wait time t_S
r_S^*
pay 0 for wait time t_0
r_F^*
pay p_F for wait time t_F
$\overline{r}$
rate of substitution

Proof

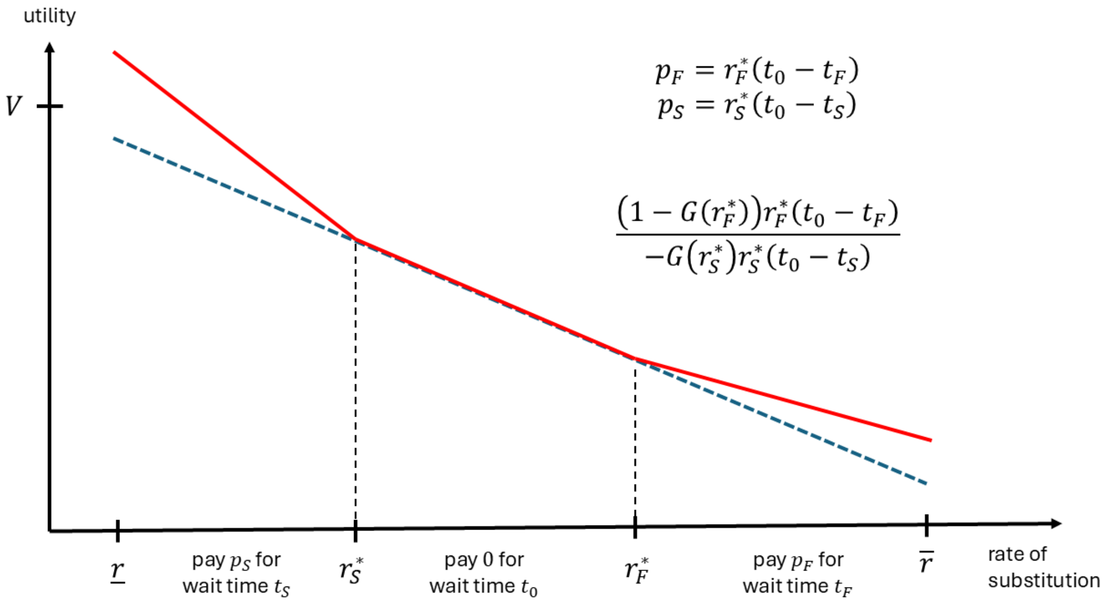

utility
V
$p_F = r_F^*(t_0 - t_F)$
$p_S = r_S^*(t_0 - t_S)$
$\frac{Rev_F}{Sub_S}$
$\underline{r}$
pay p_S for wait time t_S
r_S^*
pay 0 for wait time t_0
r_F^*
pay p_F for wait time t_F
$\bar{r}$
rate of substitution

Proof



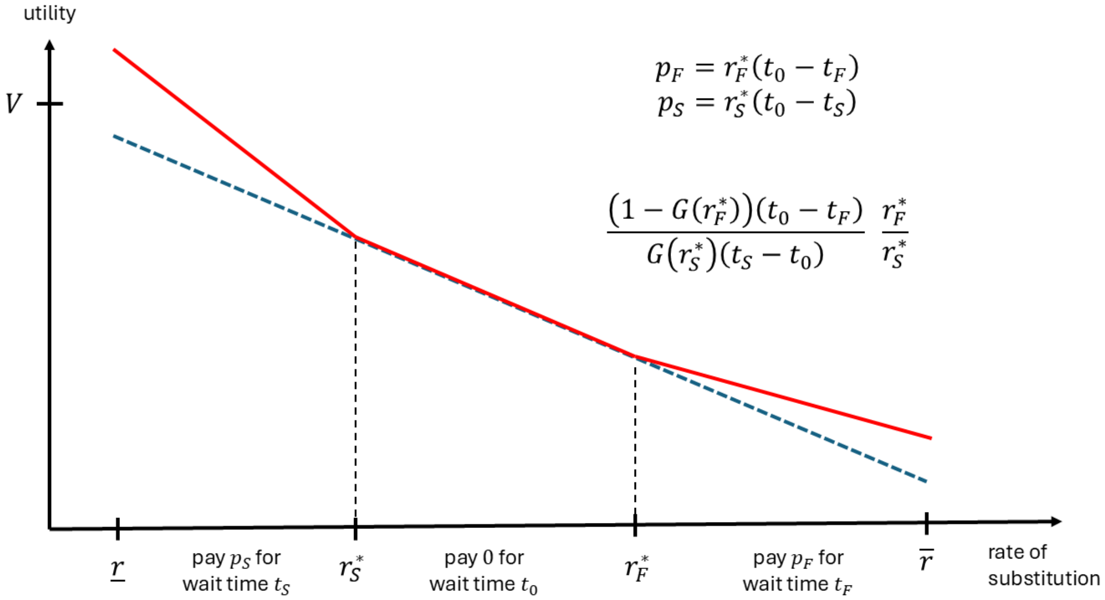
utility
V
$p_F = r_F^*(t_0 - t_F)$
$p_S = r_S^*(t_0 - t_S)$
$\frac{(1 - G(r_F^*))p_F}{-G(r_S^*)p_S}$
$\underline{r}$
pay p_S for wait time t_S
r_S^*
pay 0 for wait time t_0
r_F^*
pay p_F for wait time t_F
$\overline{r}$
rate of substitution

Proof



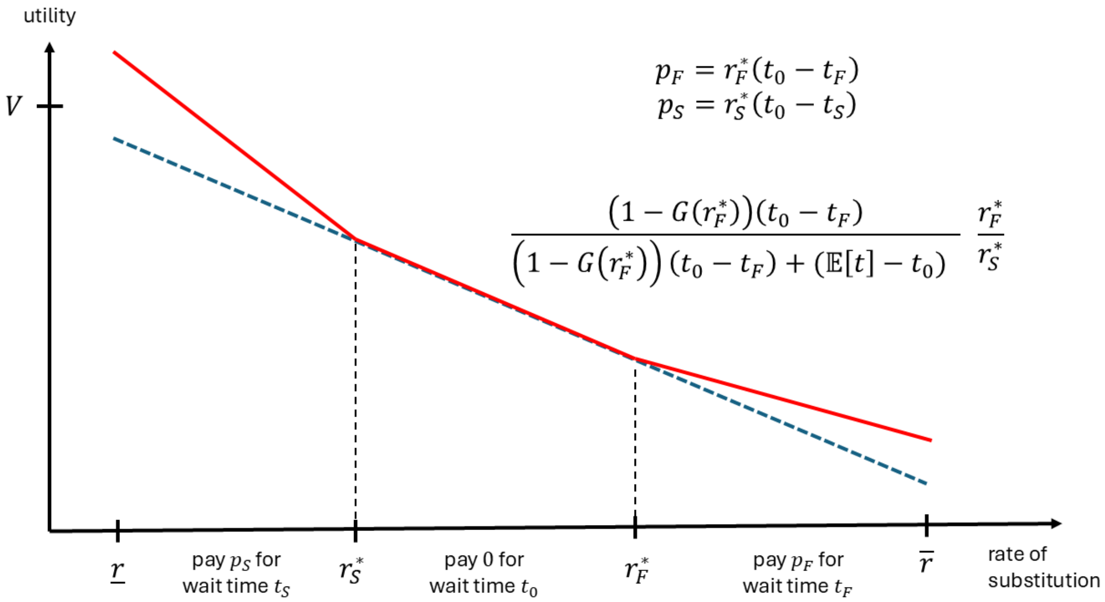
utility
V
$p_F = r_F^*(t_0 - t_F)$
$p_S = r_S^*(t_0 - t_S)$
$\frac{(1 - G(r_F^*))r_F^*(t_0 - t_F)}{-G(r_S^*)r_S^*(t_0 - t_S)}$
$\underline{r}$
pay p_S for wait time t_S
r_S^*
pay 0 for wait time t_0
r_F^*
pay p_F for wait time t_F
$\overline{r}$
rate of substitution

Proof



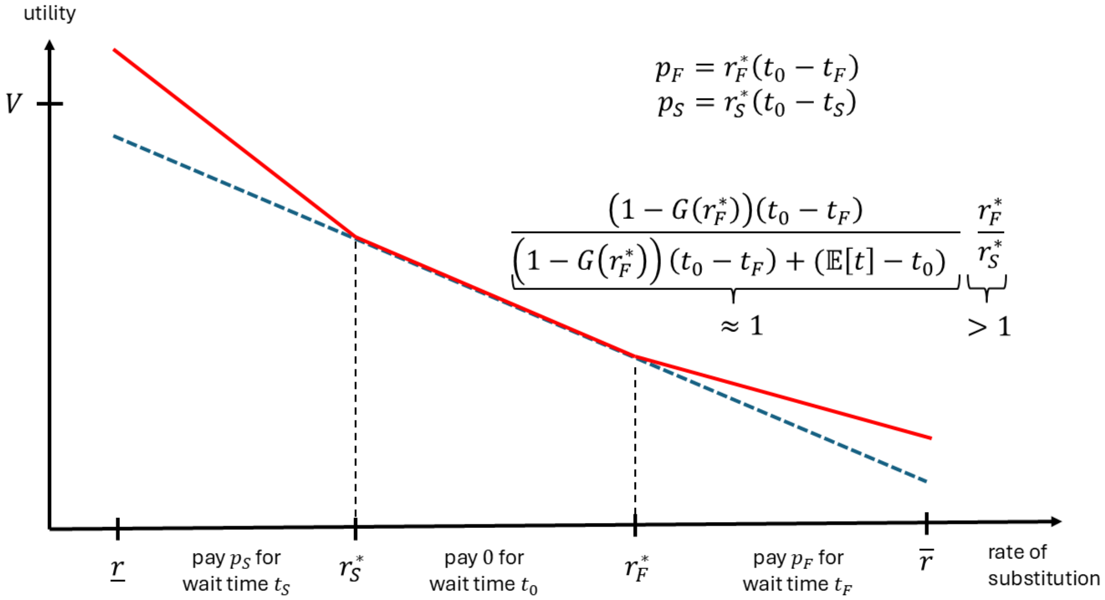
utility
V
$p_F = r_F^*(t_0 - t_F)$
$p_S = r_S^*(t_0 - t_S)$
$\frac{(1 - G(r_F^*))(t_0 - t_F)}{G(r_S^*)(t_S - t_0)}$ $\frac{r_F^*}{r_S^*}$
$\underline{r}$
pay p_S for wait time t_S
r_S^*
pay 0 for wait time t_0
r_F^*
pay p_F for wait time t_F
$\overline{r}$
rate of substitution

Proof



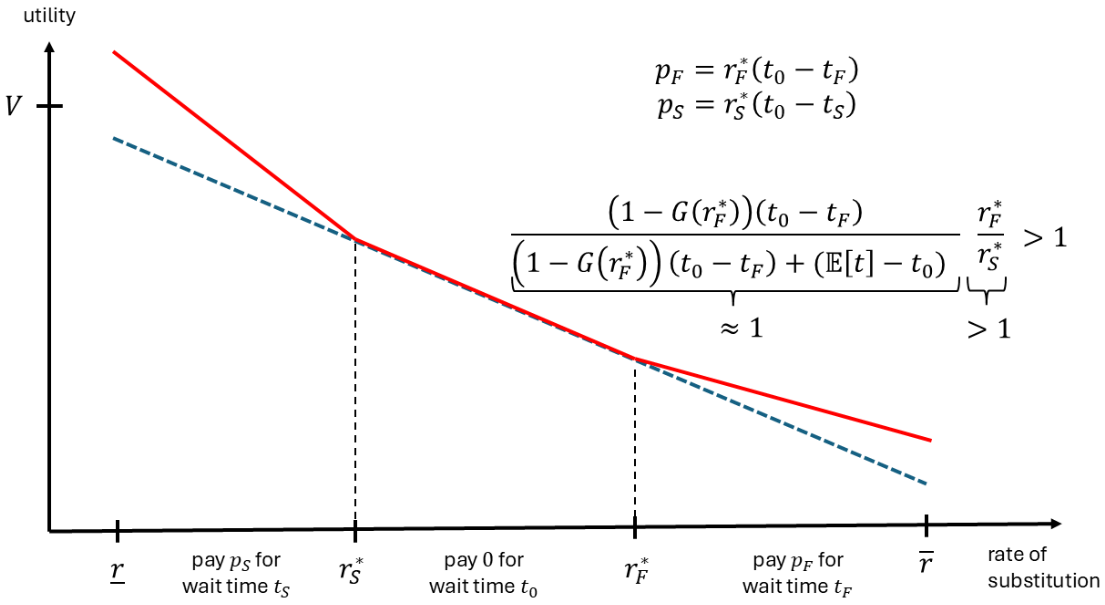
utility
V
$p_F = r_F^*(t_0 - t_F)$
$p_S = r_S^*(t_0 - t_S)$
$\frac{(1 - G(r_F^*))(t_0 - t_F)}{(1 - G(r_F^*))(t_0 - t_F) + (\mathbb{E}[t] - t_0)}$
$\frac{r_F^*}{r_S^*}$
$\underline{r}$
pay p_S for wait time t_S
r_S^*
pay 0 for wait time t_0
r_F^*
pay p_F for wait time t_F
$\overline{r}$
rate of substitution

Proof

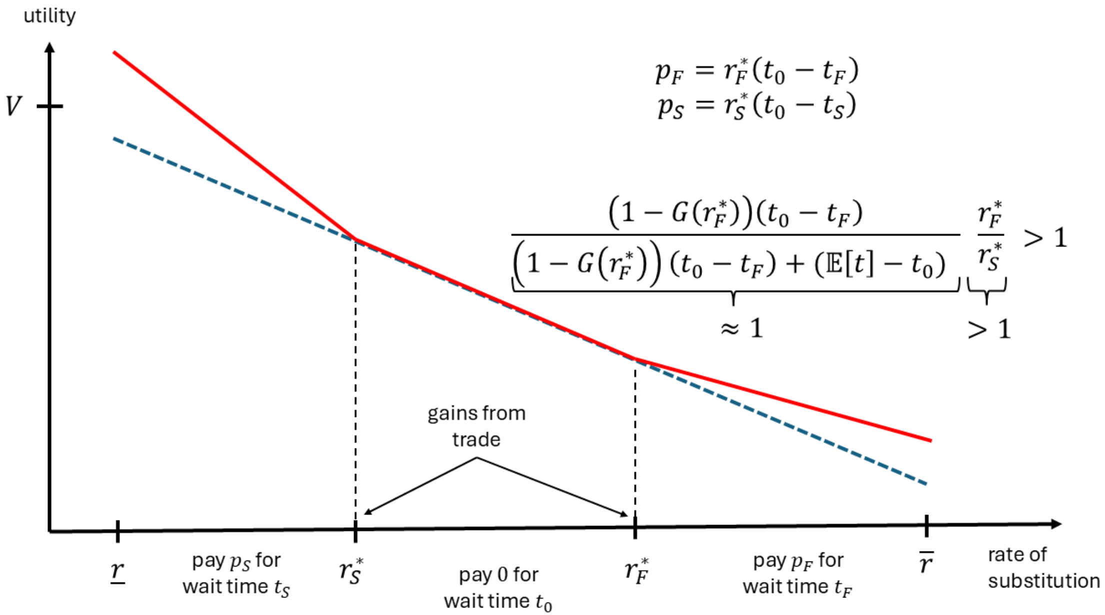


utility
V
$p_F = r_F^*(t_0 - t_F)$
$p_S = r_S^*(t_0 - t_S)$
$\frac{(1 - G(r_F^*))(t_0 - t_F)}{(1 - G(r_F^*))(t_0 - t_F) + (\mathbb{E}[t] - t_0)}$
≈ 1
$\frac{r_F^*}{r_S^*}$
> 1
$\underline{r}$
pay p_S for wait time t_S
r_S^*
pay 0 for wait time t_0
r_F^*
pay p_F for wait time t_F
$\overline{r}$
rate of substitution

Proof

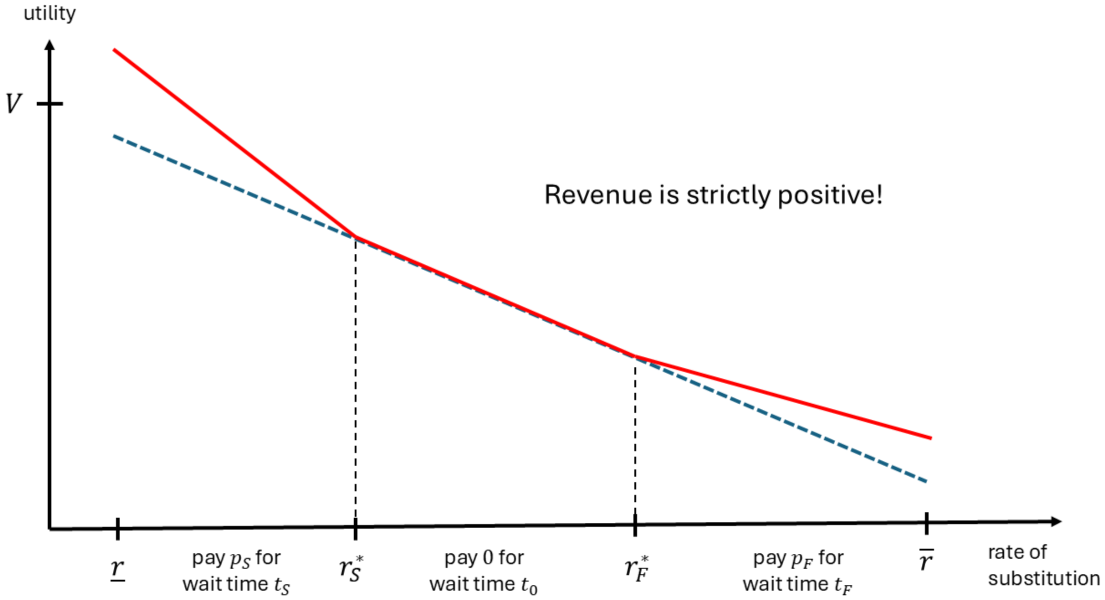

utility
V
$p_F = r_F^*(t_0 - t_F)$
$p_S = r_S^*(t_0 - t_S)$
$\underbrace{\frac{(1 - G(r_F^*))(t_0 - t_F)}{\left(1 - G(r_F^*)\right)(t_0 - t_F) + (\mathbb{E}[t] - t_0)}}_{\approx 1} \underbrace{\frac{r_F^*}{r_S^*}}_{>1} > 1$
$\underline{r}$
pay p_S for wait time t_S
r_S^*
pay 0 for wait time t_0
r_F^*
pay p_F for wait time t_F
$\bar{r}$
rate of substitution

Proof



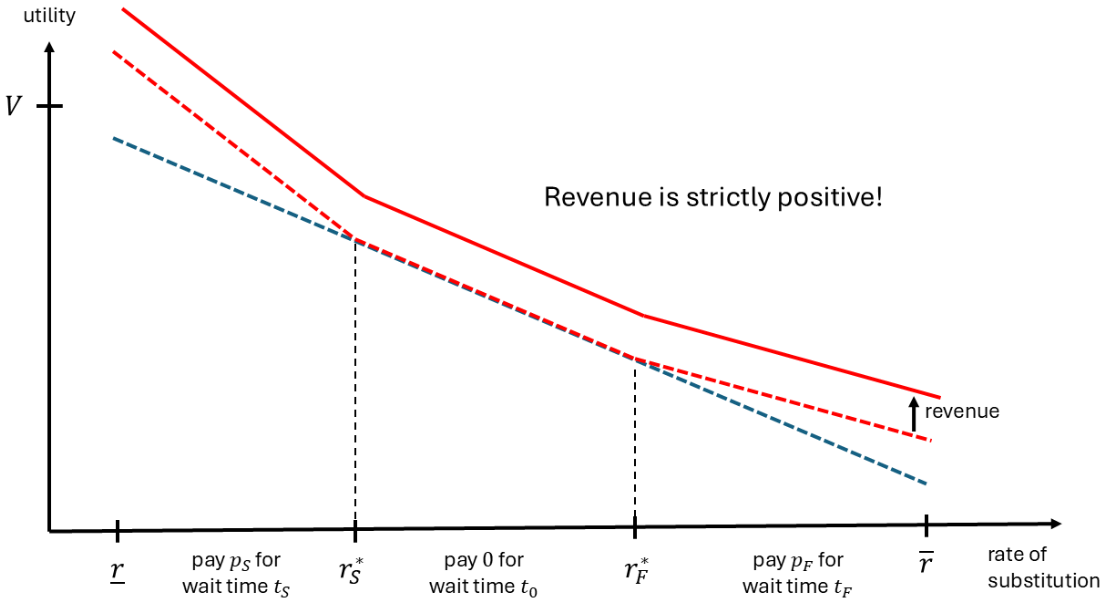
utility
V
$p_F = r_F^*(t_0 - t_F)$
$p_S = r_S^*(t_0 - t_S)$
$\underbrace{\frac{(1-G(r_F^*))(t_0 - t_F)}{(1-G(r_F^*))(t_0 - t_F) + (\mathbb{E}[t] - t_0)}}_{\approx 1} \underbrace{\frac{r_F^*}{r_S^*}}_{>1} > 1$
gains from trade
$\underline{r}$
pay p_S for wait time t_S
r_S^*
pay 0 for wait time t_0
r_F^*
pay p_F for wait time t_F
$\overline{r}$
rate of substitution

Proof



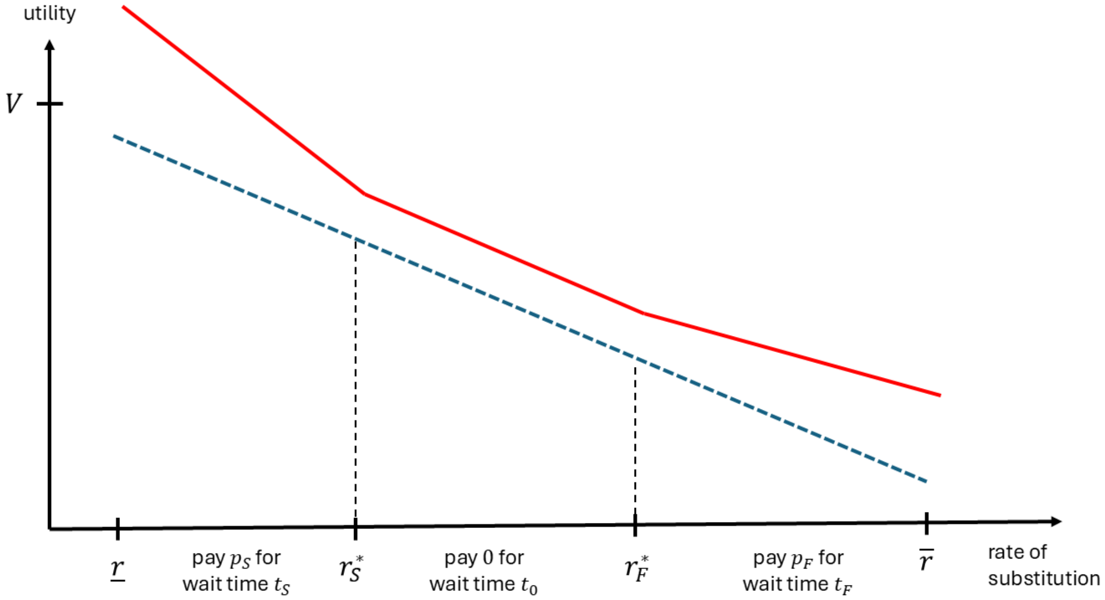
utility
V
Revenue is strictly positive!
$\underline{r}$
pay p_S for
wait time t_S
r_S^*
pay 0 for
wait time t_0
r_F^*
pay p_F for
wait time t_F
$\overline{r}$
rate of
substitution

Proof



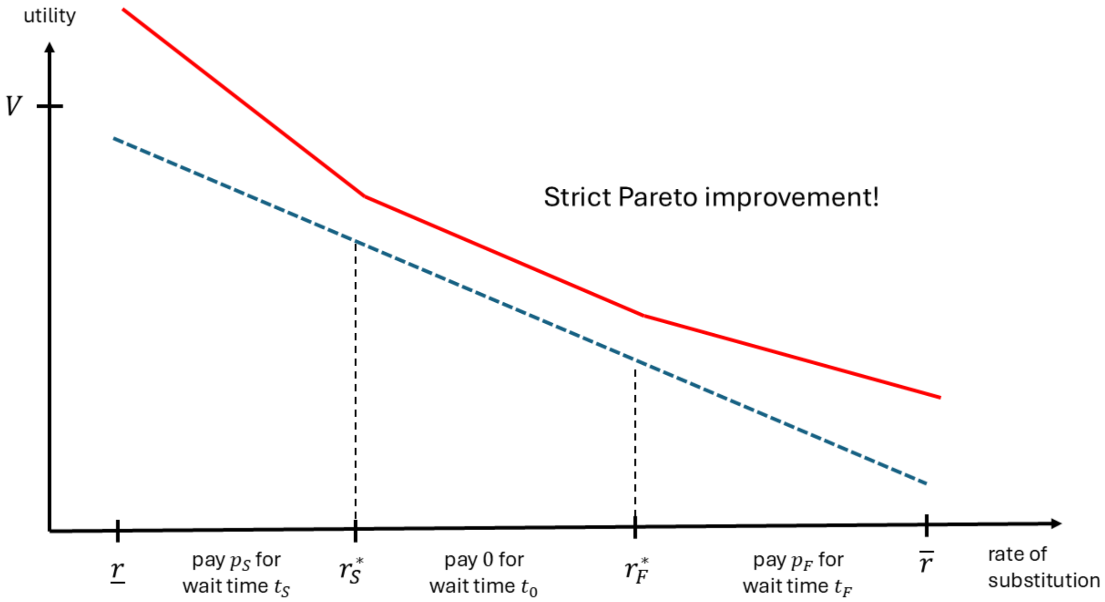
utility
V
Revenue is strictly positive!
revenue
$\underline{r}$
pay p_S for
wait time t_S
r_S^*
pay 0 for
wait time t_0
r_F^*
pay p_F for
wait time t_F
$\overline{r}$
rate of
substitution

Proof



utility
V
$\underline{r}$
pay p_S for
wait time t_S
r_S^*
pay 0 for
wait time t_0
r_F^*
pay p_F for
wait time t_F
$\overline{r}$
rate of
substitution

Proof



utility
V
Strict Pareto improvement!
$\underline{r}$
pay p_S for
wait time t_S
r_S^*
pay 0 for
wait time t_0
r_F^*
pay p_F for
wait time t_F
$\bar{r}$
rate of
substitution

Concluding Remarks

Applications

We consider various applications:

#1. lane pricing (running example);

#2. waiting in line for a service; and

#3. price discrimination (à la Mussa and Rosen, 1978) with a capacity constraint.

Extensions

In the baseline model, **can we Pareto improve** on a two-tier priority system?

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Unfortunately, the answer in general is "no."

- Take the running example with $N = 2$; for any choice of m, there are welfare functions for which **assortative matching is optimal** (Akbarpour ⓡ Dworczak ⓡ Kominers, 2024).

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- $\rightsquigarrow$ Take the running example with $N = 2$; for any choice of m, there are welfare functions for which **assortative matching is optimal** (Akbarpour ⓡ Dworczak ⓡ Kominers, 2024).

This means that **some 2-tier priority systems** are optimal for some welfare weights, hence **on the Pareto frontier**!

Intuition: a 2-tier priority system can be very good for agents with low and high MRS.

Extensions

We argued that there exists a Pareto improvement, but **what mechanism should we use**?

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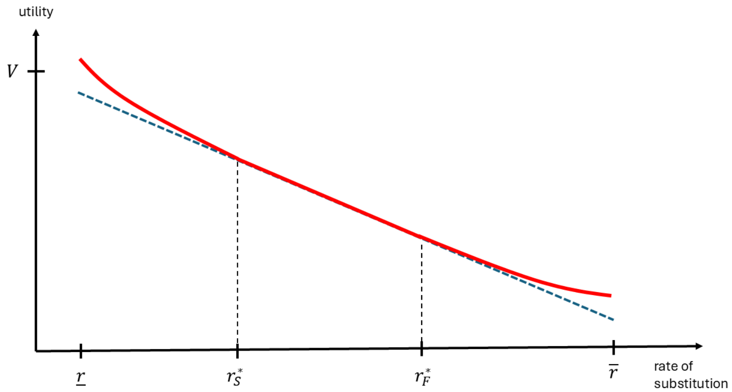
One approach is to maximize a particular welfare function subject to Pareto improvement.

In the waiting in line example:



Concluding Remarks

In a simple model of **priority pricing**, a **Pareto improvement** over an unpriced benchmark is possible with **3 priority tiers**, but not with 2.

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A concrete **policy implication**:

> Create at least 3 options, with the middle option being free and replicating the wait time in the unpriced benchmark.

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In a simple model of **priority pricing**, a **Pareto improvement** over an unpriced benchmark is possible with **3 priority tiers**, but not with 2.

A concrete **policy implication**:

> Create at least 3 options, with the middle option being free and replicating the wait time in the unpriced benchmark.

Next steps?

- Conditions for Pareto improvement over an arbitrary mechanism.
- Robustness to extensive margin and richer preferences.
- Empirical tests.

Concluding Remarks

$$3 > 2$$

References