

Topping Up and Optimal Redistribution

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Motivation

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 - ▶ food stamps (e.g., SNAP); and
 - ▶ public housing programs (e.g., public housing developments, LIHTC).

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Topping up is often a design choice of in-kind programs and affects screening:

- ▶ Unlike public housing units, housing vouchers would allow topping up (if desired).
- ▶ While recipients benefit, topping up weakens screening by expanding participation.

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“Programs with or without the possibility of topping up have different welfare properties. Currently, there are no general results regarding the merits of the two. [...] Nor are there any general results on the characterization of optimal public provision policies, targeted or universal.”

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This paper: How does topping up affect the optimal design of redistribution programs?

↪ **Subsidies** \neq **money**: we restrict social planner's ability to use lump-sum transfers.

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Main Results:

- #1. **Extensive margin:** how does topping up impact **when** to redistribute?
- #2. **Intensive margin:** how does topping up impact **how** to redistribute?

Model

Setup

- ▶ There is a unit mass of risk-neutral consumers demanding some quantity of a good. The good is produced competitively at a constant marginal cost, $c > 0$.

Setup

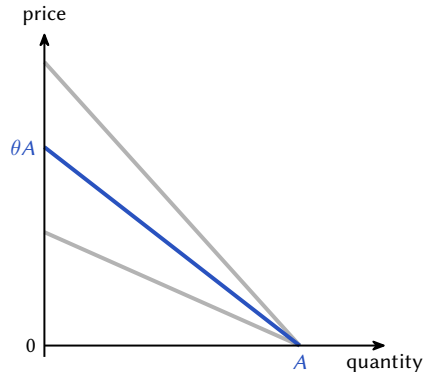
- ▶ There is a unit mass of risk-neutral consumers demanding some quantity of a good. The good is produced competitively at a constant marginal cost, $c > 0$.
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- ▶ Each consumer derives utility $\theta v(q) - t$ from quantity $q \in [0, A]$ at total price t .

Assume that $v : [0, A] \rightarrow \mathbb{R}$ is increasing and strictly concave (with $v' > 0$ and $v'' < 0$).

Example: $v(q) = Aq - \frac{1}{2}q^2$ (linear demand).

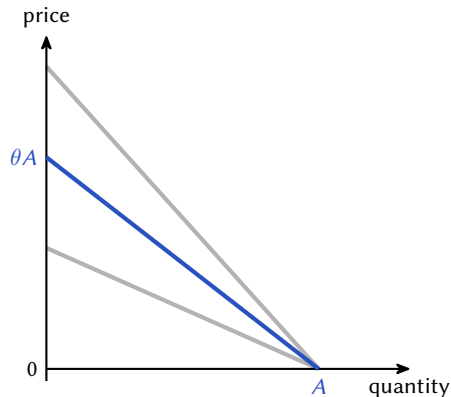


Laissez-Faire Equilibrium

- ▶ Since the private market is perfectly competitive, price = marginal cost = c .
- ▶ Each consumer solves

$$U^{\text{LF}}(\theta) := \max_{q \in [0, A]} [\theta v(q) - cq].$$

Since v is strictly concave, there is a unique maximizer $q^{\text{LF}}(\theta)$, assumed to be interior.

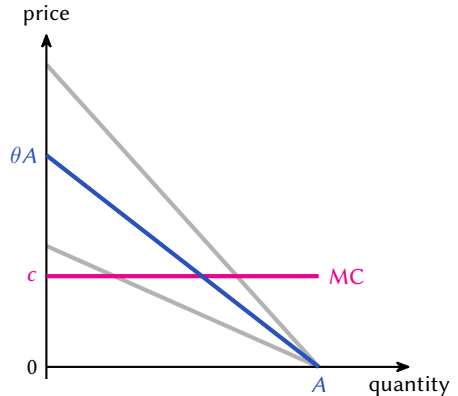


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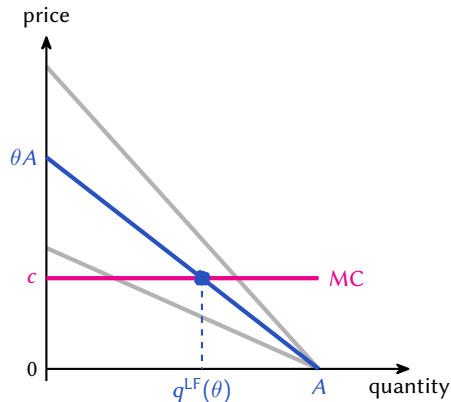


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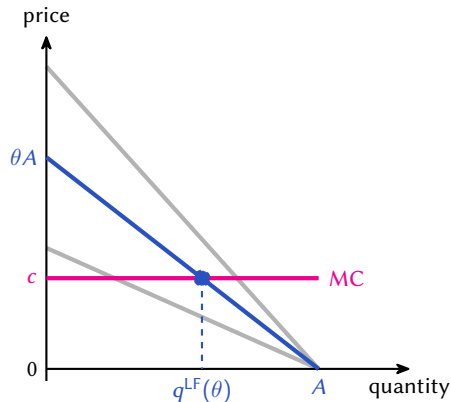
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▶ Assumption of constant MC can be relaxed.

- Upward-sloping supply: Kang (2023).
Eqm. effects \Rightarrow alt. redistribution channel.
- **This paper:** participation constraints.



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- ▶ Equivalently, the social planner chooses a **direct mechanism** (q, t) , consisting of:
 - the **allocation function** $q : [\underline{\theta}, \bar{\theta}] \rightarrow [0, A]$ denoting *total* consumption by consumer;
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 - the **payment function** $t : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}_+$ denoting *total* payment by consumer.
- ▶ We focus on **in-kind** programs: $t(\cdot) \geq 0 \rightsquigarrow$ no lump-sum transfers to consumers.
However, we allow for lump-sum transfers to consumers “outside of the mechanism.”

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The social planner maximizes **weighted total surplus**, which consists of:

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- ▶ Total profit

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 - $\rightsquigarrow \omega(\theta)$: expected social value of giving him one unit of money.
 - \rightsquigarrow Monotonicity of ω captures the correlation between demand and redistributive priority.

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For any given mechanism (q, t) , the weighted total surplus can be written as:

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} + \alpha \underbrace{[t(\theta) - cq(\theta)]}_{\text{total profit}} \right] dF(\theta).$$

Mechanism Design

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subject to

- ▶ incentive compatibility, $\theta \in \arg \max_{\hat{\theta} \in [\underline{\theta}, \bar{\theta}]} [\theta v(q(\hat{\theta})) - t(\hat{\theta})], \quad \forall \theta \in [\underline{\theta}, \bar{\theta}]; \quad (\text{IC})$
- ▶ no lump-sum transfers, $t(\theta) \geq 0, \quad \forall \theta \in [\underline{\theta}, \bar{\theta}]; \quad (\text{LS})$
- ▶ individual rationality, $\theta v(q(\theta)) - t(\theta) \geq U^{\text{LF}}(\theta), \quad \forall \theta \in [\underline{\theta}, \bar{\theta}]; \quad (\text{IR})$
- ▶ topping up, $q(\theta) \geq q^{\text{LF}}(\theta), \quad \forall \theta \in [\underline{\theta}, \bar{\theta}]. \quad (\text{TU})$

Related Work

- ▶ **Redistributive Mechanism Design.** Che, Gale and Kim (2013); Condorelli (2013); Dworzak (r) Kominers (r) Akbarpour (2021); Akbarpour (r) Dworzak (r) Kominers (2022); Akbarpour (r) Dworzak (r) Kominers (2024); Pai and Strack (2024); Vairo (2025).
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- ▶ **“Partial” Mechanism Design.** Philippon and Skreta (2012); Tirole (2012); Fuchs and Skrzypacz (2015); Dworzak (2020); Loertscher and Muir (2022); Kang and Muir (2022); Kang (2023).
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- ▶ **Methodological Tools in Mechanism Design.**
 - Generalized ironing: Toikka (2011).
 - Lagrangian approach: Amador, Werning and Angeletos (2006); Amador and Bagwell (2013).
 - Type-dependent outside options: Jullien (2000), Dworzak and Muir (2024).
 - Majorization constraints (in a concave program): Kleiner, Moldovanu and Strack (2021).
 - ~ **This paper:** majorization constraints due to type-dependent outside options (in a convex program).

Related Work

► Public Finance.

Theory: Nichols and Zeckhauser (1982), Blackorby and Donaldson (1988), Coate (1989), Besley and Coate (1991), Coate, Johnson and Zeckhauser (1994), Gahvari and Mattos (2007).

Empirics: Baum-Snow and Marion (2009), Diamond and McQuade (2019), van Dijk (2019), Dinerstein, Neilson and Otero (2020), Atal, Cuesta, González and Otero (2021), Dinerstein and Smith (2021), Jiménez-Hernández and Seira (2021), Handbury and Moshary (2021).

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- As Doligalski (r) Dworzak (r) Akbarpour (r) Kominers (2025) write:

“Several decades after the original work of Atkinson and Stiglitz (1976), conventional economic wisdom seems to have embraced the idea of using income taxes rather than goods market interventions to redistribute; it would appear that this intuition needs to be revisited, and more research is needed to understand whether it constitutes good policy advice under realistic scenarios.”

Topping Up vs. No Topping Up

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② **How to optimally redistribute?** (*intensive-margin differences*).

$N + S$ conditions for optimal mechanisms to coincide \rightsquigarrow topping up distorts screening.

① When to Optimally Redistribute?

Theorem 1. The optimal mechanism strictly improves on the laissez-faire outcome iff:

$$\begin{cases} \max_{\hat{\theta} \in [\underline{\theta}, \bar{\theta}]} \mathbf{E}[\omega(\theta) \mid \theta \geq \hat{\theta}] > \alpha, & \text{if topping up is allowed,} \\ \max \omega > \alpha, & \text{if topping up is not allowed.} \end{cases}$$

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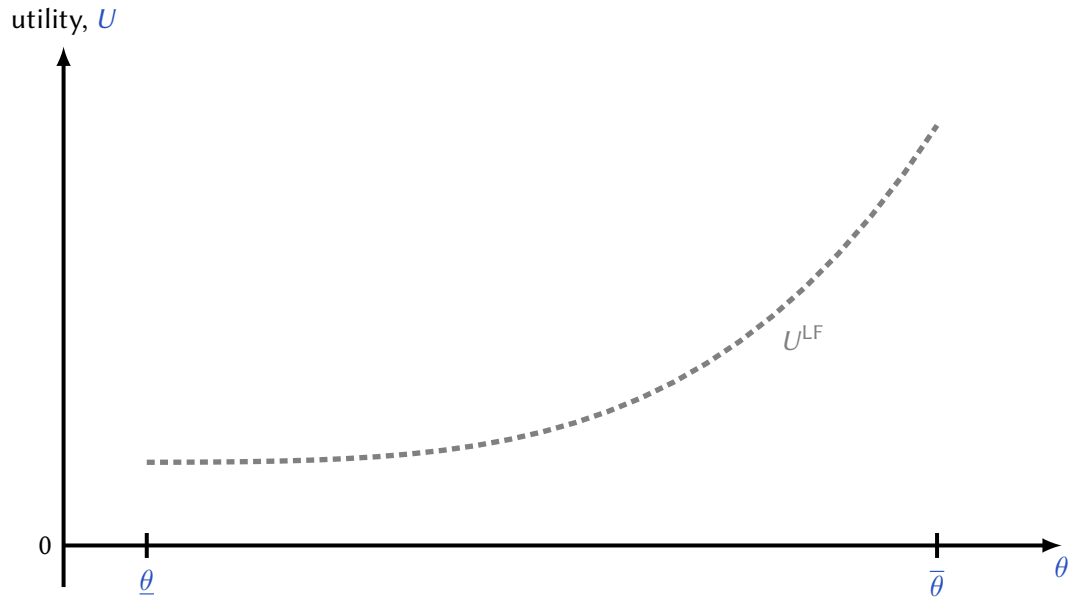
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- ▶ **Negative correlation.** If ω is decreasing, $\max_{\hat{\theta} \in [\underline{\theta}, \bar{\theta}]} \mathbf{E}[\omega(\theta) \mid \theta \geq \hat{\theta}] = \mathbf{E}[\omega] < \max \omega$.

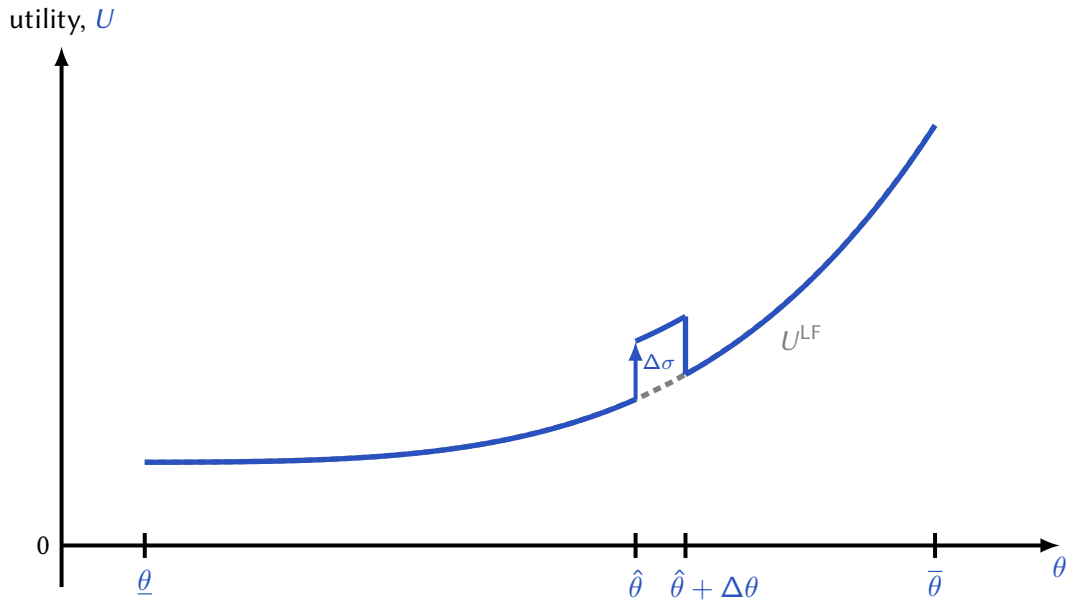
Proof Sketch of Theorem 1



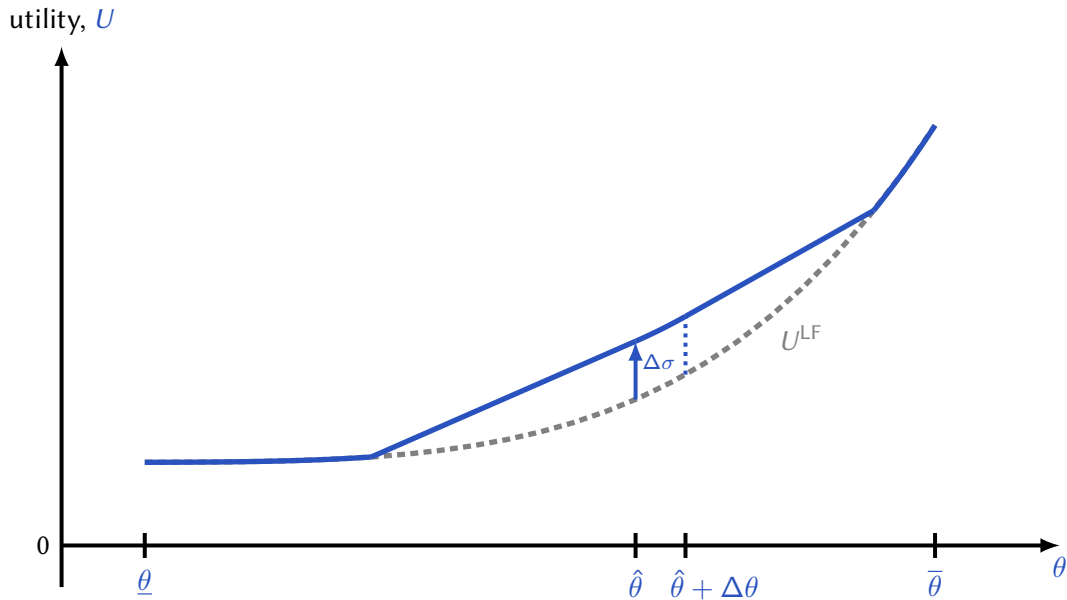
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Proof Sketch of Theorem 1: No Topping Up



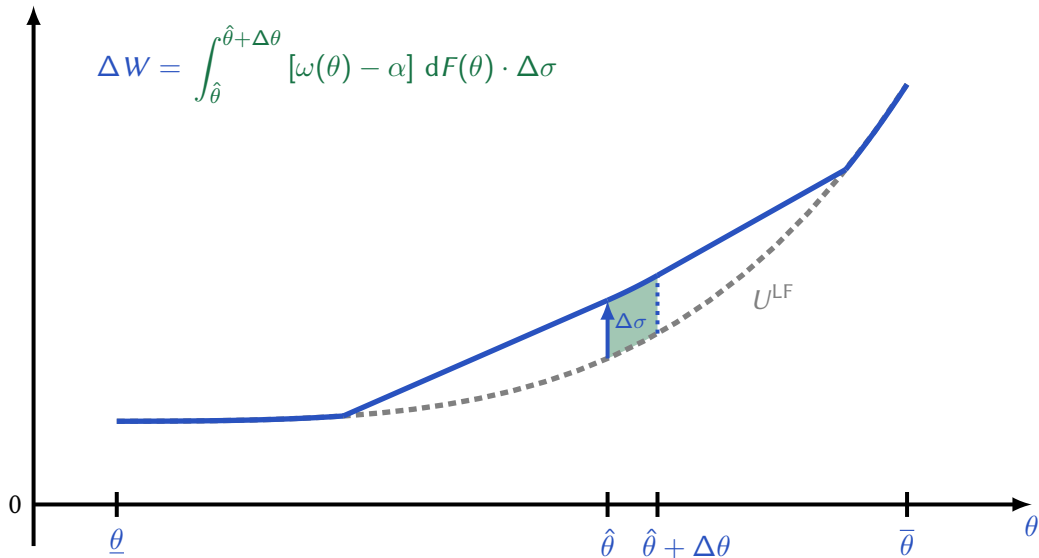
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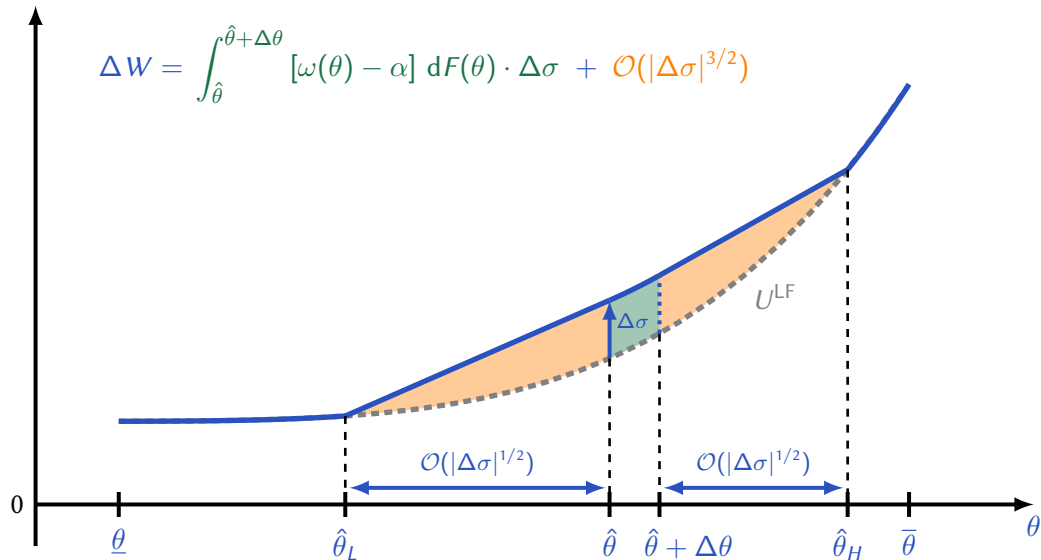
$$\Delta W = \int_{\hat{\theta}}^{\hat{\theta} + \Delta\theta} [\omega(\theta) - \alpha] dF(\theta) \cdot \Delta\sigma$$



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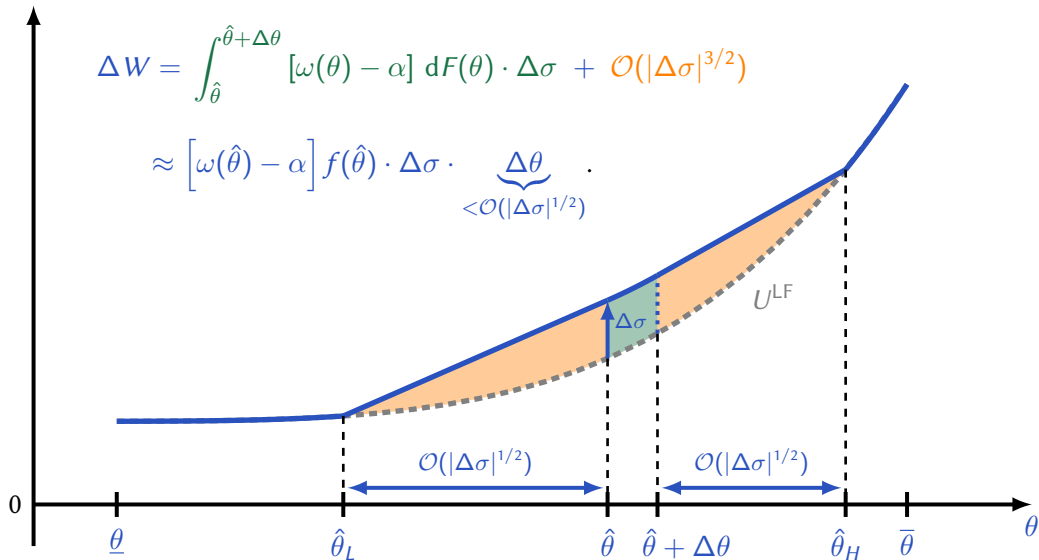


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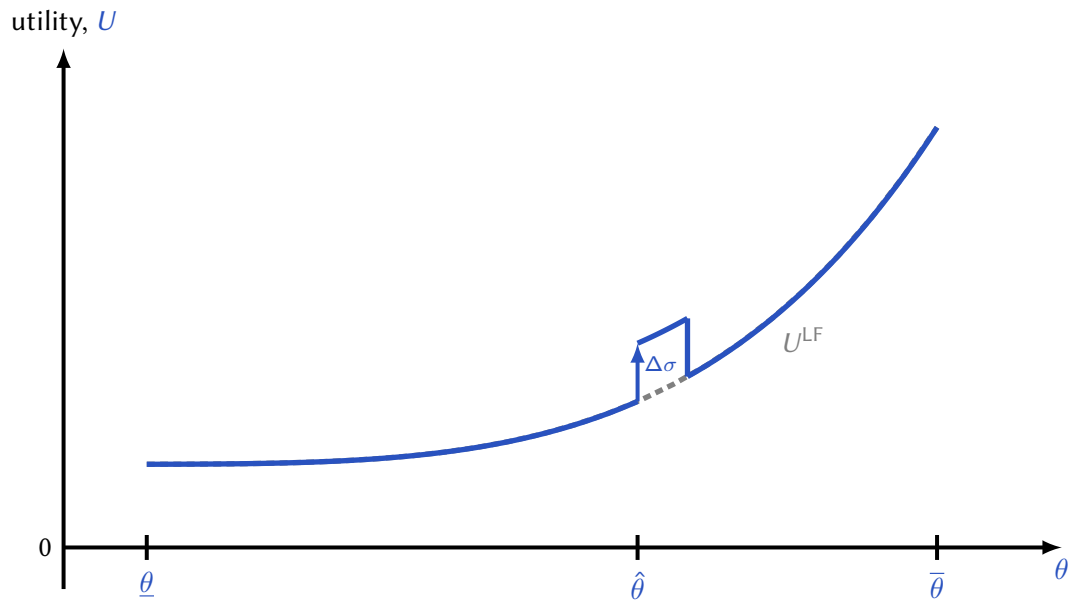
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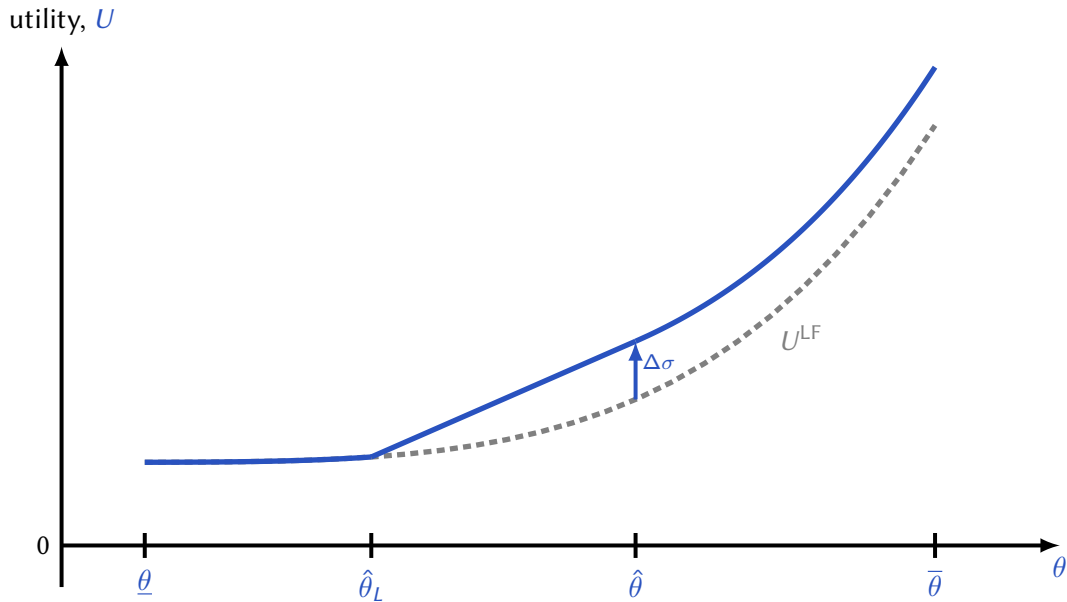
$$\approx [\omega(\hat{\theta}) - \alpha] f(\hat{\theta}) \cdot \Delta\sigma \cdot \underbrace{\Delta\theta}_{< \mathcal{O}(|\Delta\sigma|^{1/2})}$$



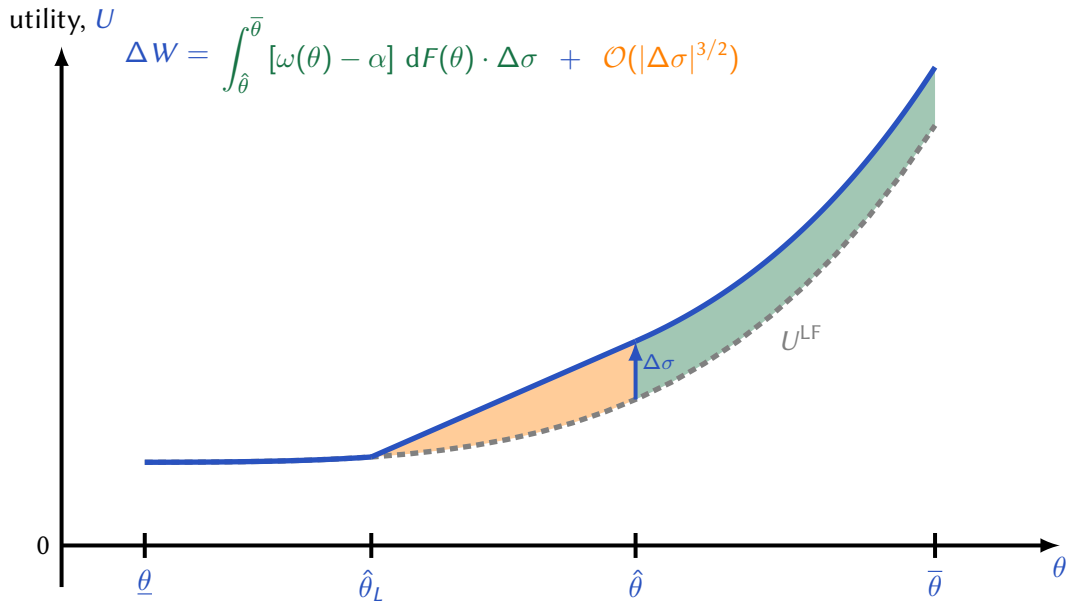
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Corollary. Topping up changes intervention decision iff $\max \omega > \alpha \geq \max_{\hat{\theta}} \mathbf{E}[\omega(\theta) \mid \theta \geq \hat{\theta}]$.

② How to Optimally Redistribute?

Theorem 2. Define

$$J_{\mu}(\theta) := \theta + \frac{\mu \underline{\theta} \cdot \delta_{\underline{\theta}}(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} [\alpha - \omega(s)] dF(s)}{\alpha f(\theta)} \quad \text{and} \quad \theta^L := \min \left\{ \hat{\theta} \in [\underline{\theta}, \bar{\theta}] : \int_{\hat{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s) \geq 0 \right\}.$$

Let $\mu^* := (\mathbf{E}[\omega] - \alpha)_+$. The optimal mechanism with and without topping up coincide iff:

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Topping up can distort screening on the intensive margin, even conditional on intervention.

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$$J_{\mu}(\theta) := \theta + \frac{\mu \underline{\theta} \cdot \delta_{\underline{\theta}}(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} [\alpha - \omega(s)] dF(s)}{\alpha f(\theta)} \quad \text{and} \quad \theta^L := \min \left\{ \hat{\theta} \in [\underline{\theta}, \bar{\theta}] : \int_{\hat{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s) \geq 0 \right\}.$$

Let $\mu^* := (\mathbf{E}[\omega] - \alpha)_+$. The optimal mechanism with and without topping up coincide iff:

$$\begin{cases} \overline{J_{\mu^*} |_{[\theta^L, \bar{\theta}]}} \geq \theta, & \text{for all } \theta^L \leq \theta \leq \bar{\theta}, \\ \omega(\theta) \leq \alpha, & \text{for all } \underline{\theta} \leq \theta < \theta^L. \end{cases}$$

Topping up can distort screening on the intensive margin, even conditional on intervention.

Next: how topping up affects **optimal redistribution** for positive vs. negative correlation.

Positive Correlation

Optimal Mechanisms: Positive Correlation

Theorem 3. When correlation is positive (i.e., ω is increasing), topping up **does not affect** optimal redistribution: the optimal mechanisms with and without topping up coincide.

Intuition:

- ▶ Optimal mechanism distorts consumption \uparrow for consumers with high θ (= high ω).
- ▶ This is “well-targeted”: benefits accrue to consumers with even higher θ (= higher ω).
- ▶ If not for topping up, the social planner would provide the “topped-up” units herself.

Optimal Mechanisms: Positive Correlation

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Examples: childcare, disability care, public transit, basic staples like coarse bread/cassava.

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Examples: childcare, disability care, public transit, basic staples like coarse bread/cassava.

The *Public Expenditure Handbook* published by the IMF recommends that:

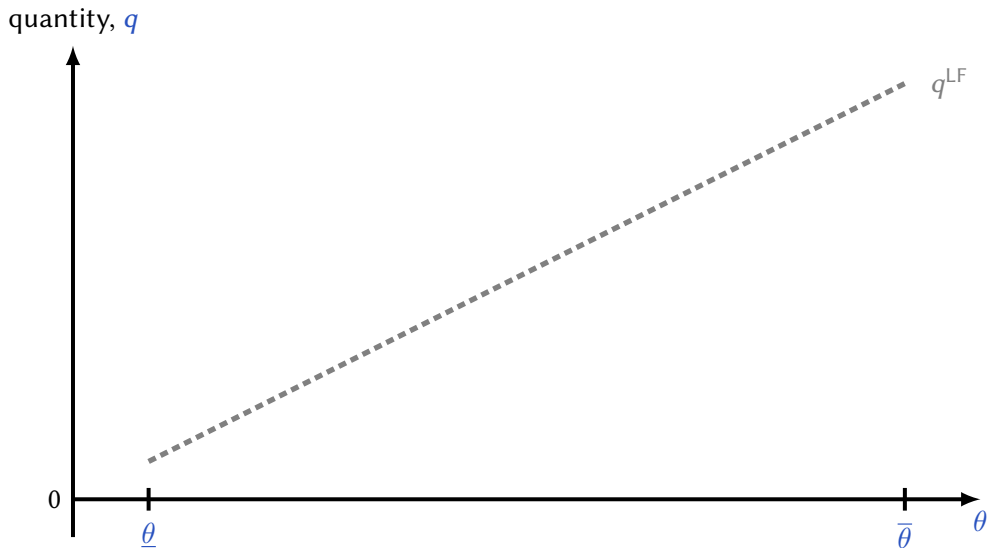
“[m]arketed goods with a negative income elasticity (i.e., inferior goods) are ideal candidates for a redistributive subsidy.”

Theorem 3 complements the “self-targeting” explanation: no need to prevent topping up.

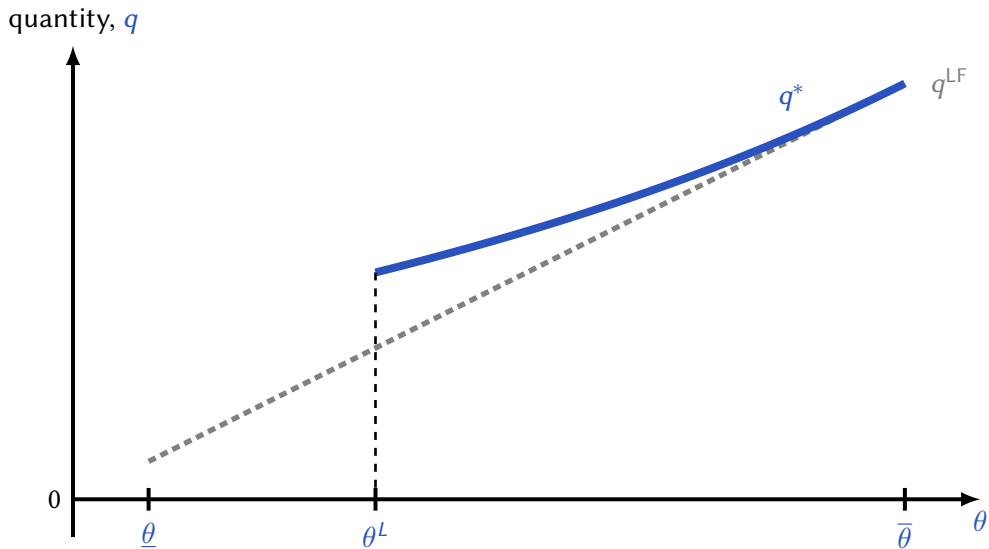
Characterization of Optimal Mechanisms: $E[\omega] > \alpha$



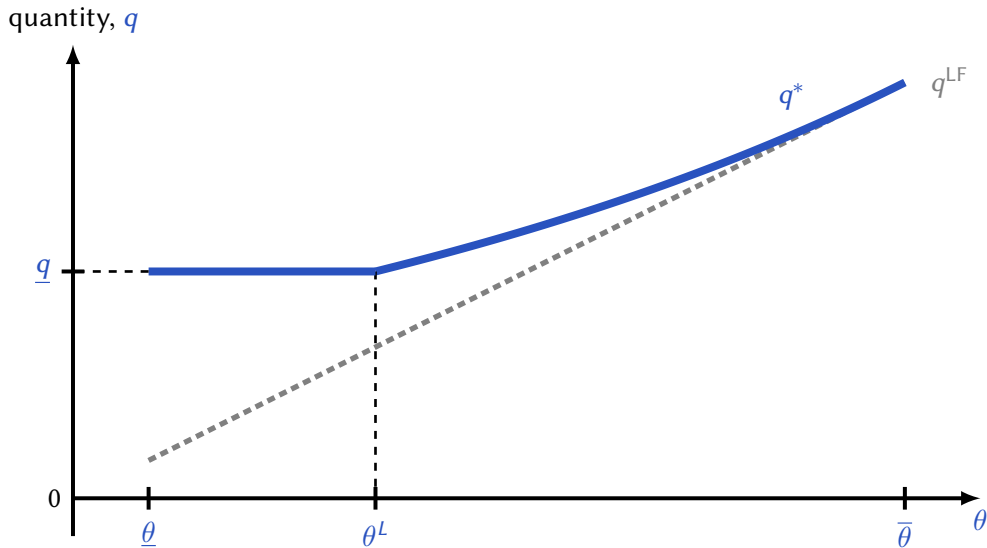
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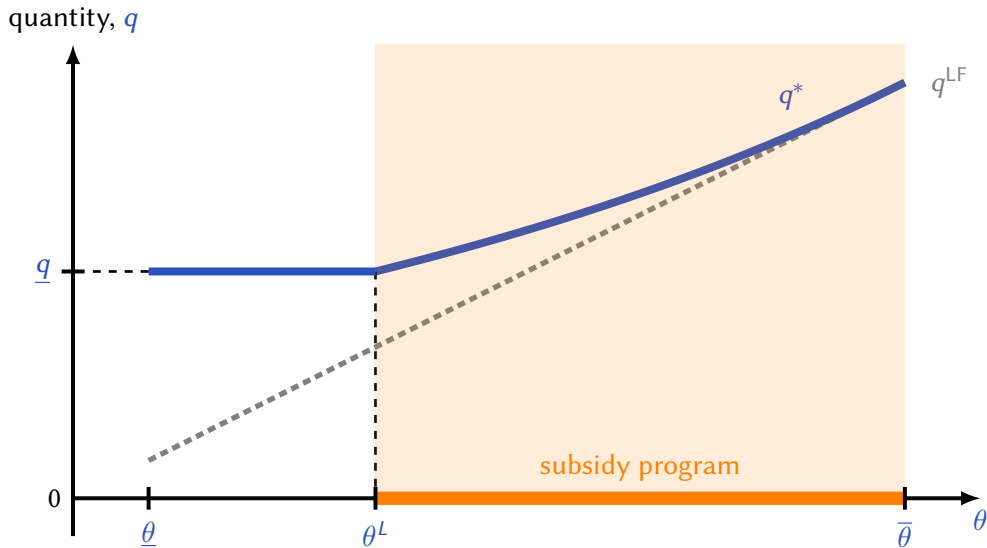
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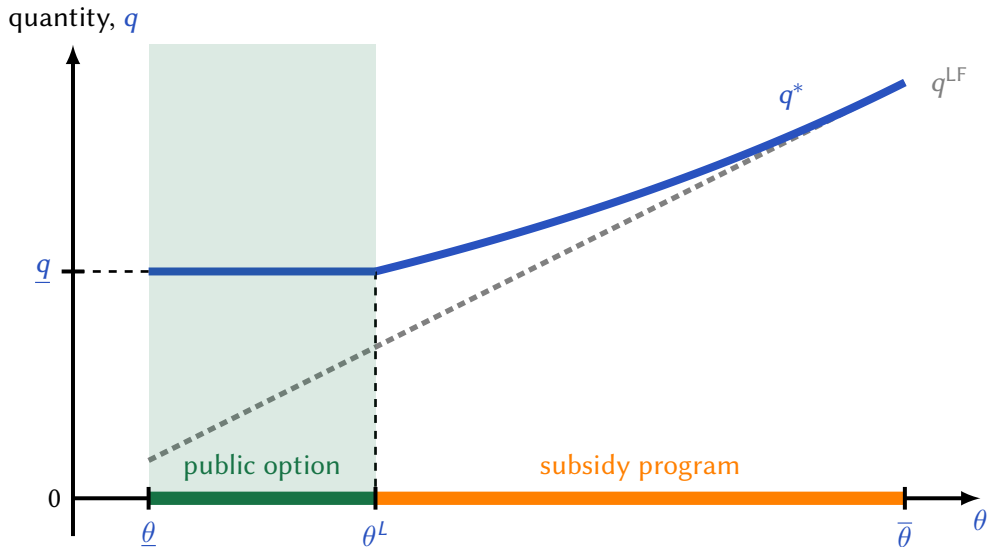
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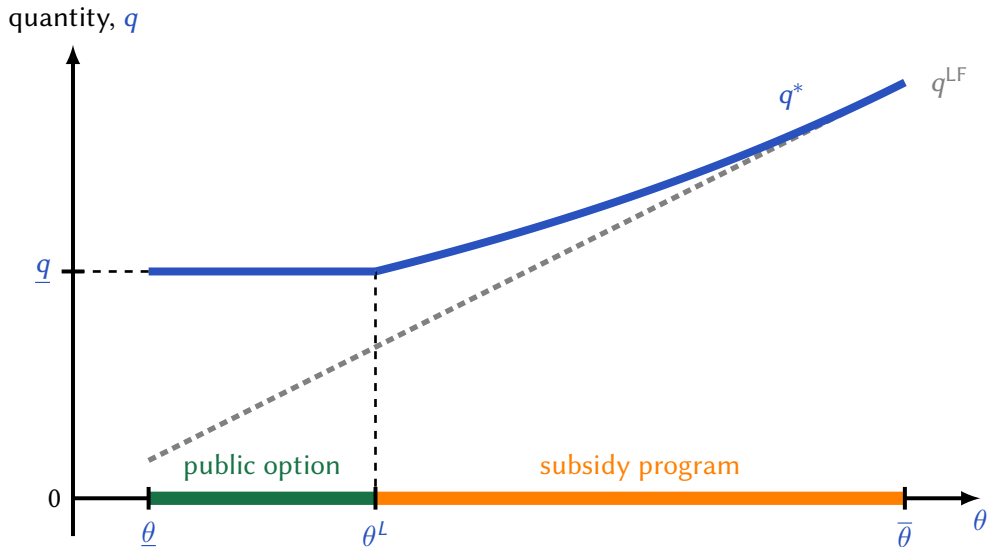
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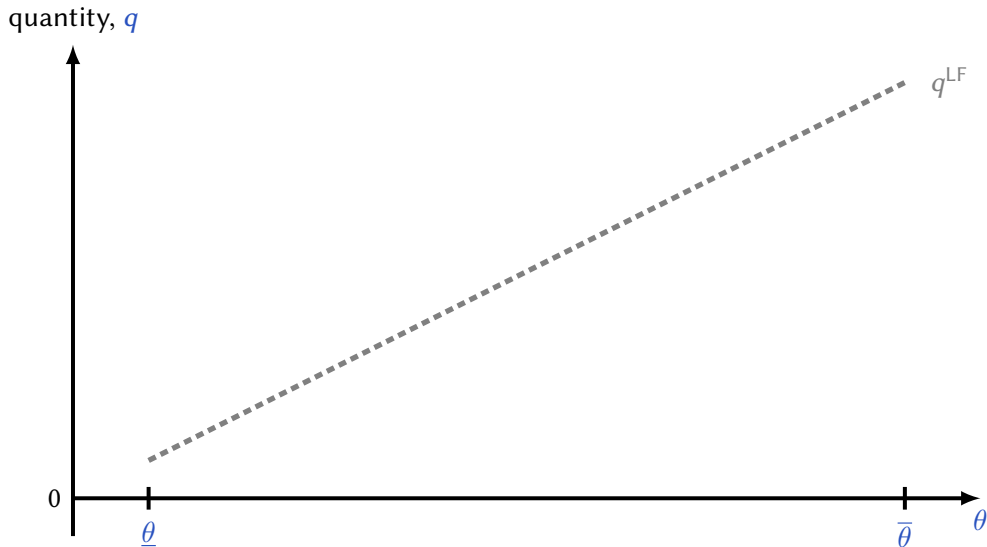
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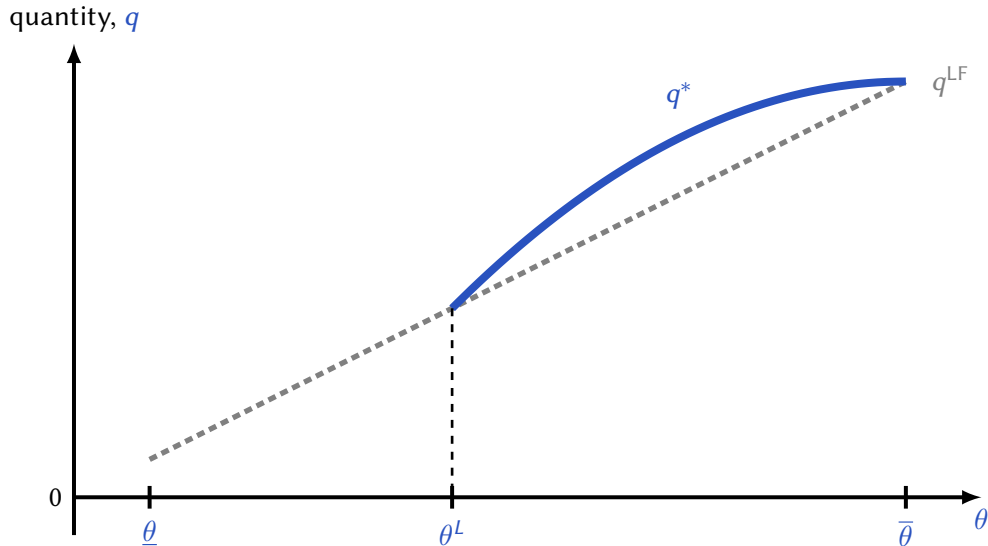
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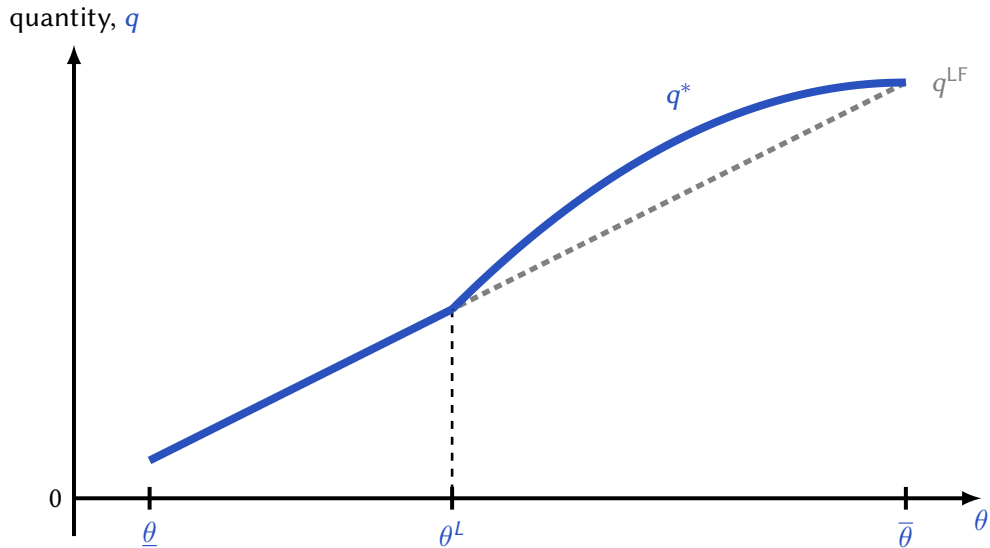
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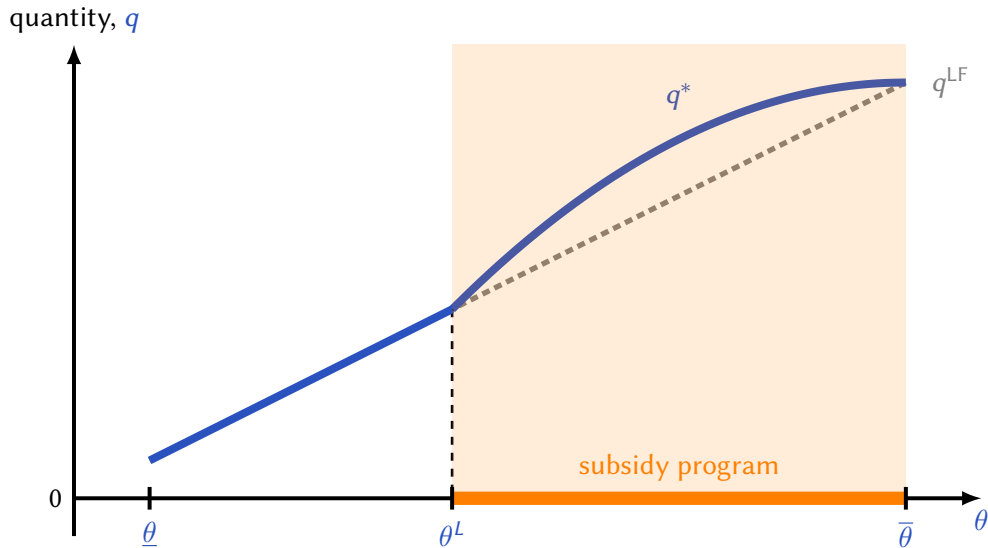
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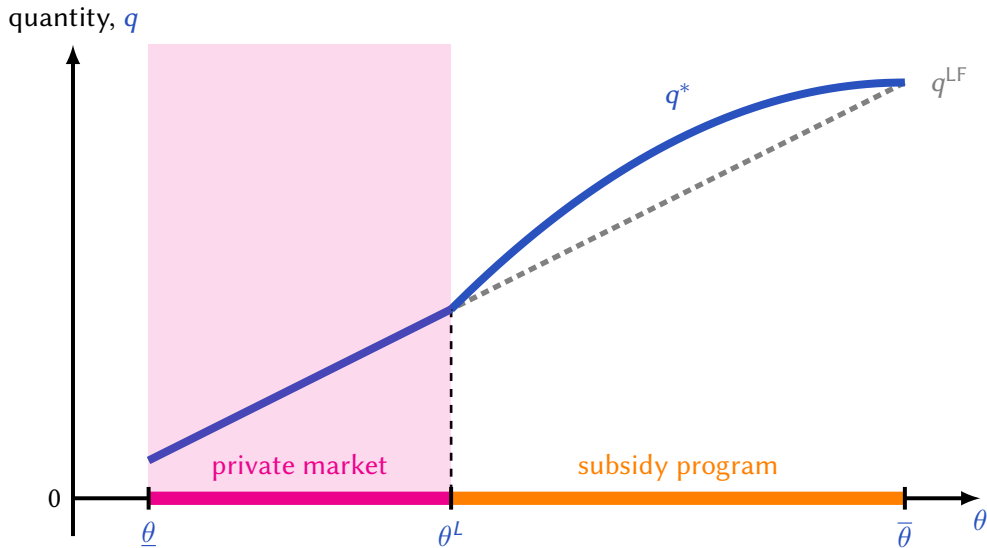
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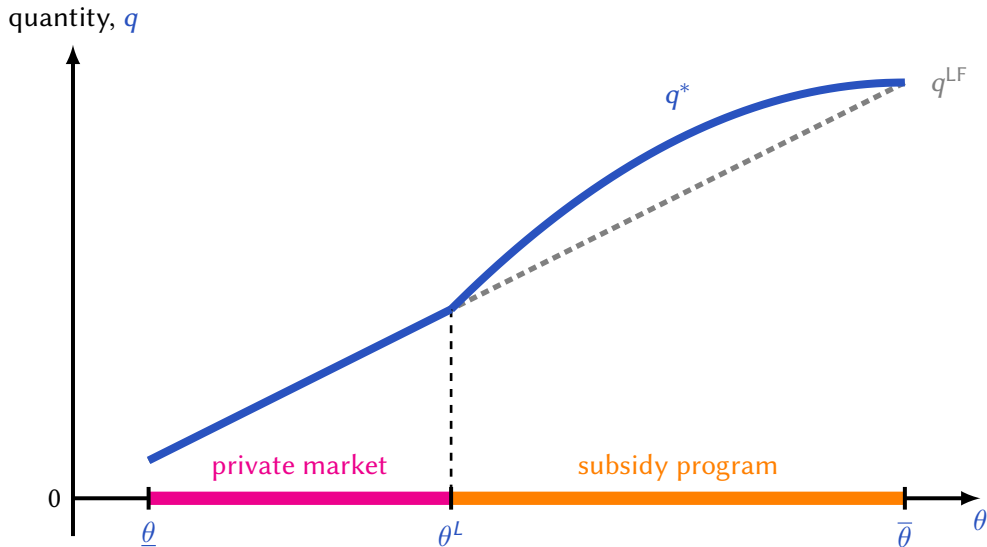
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Optimal Mechanisms: Positive Correlation

Theorem 3. When correlation is positive (i.e., ω is increasing), topping up **does not affect** optimal redistribution: the optimal mechanisms with and without topping up coincide.

If $E[\omega] > \alpha$: public option subsidy program demand

If $E[\omega] \leq \alpha$: private market subsidy program demand

Optimal Mechanisms: Positive Correlation

Theorem 3. When correlation is positive (i.e., ω is increasing), topping up **does not affect** optimal redistribution: the optimal mechanisms with and without topping up coincide.

In particular, for any $\mu \geq 0$, define

$$q_\mu(\theta) := D(c, \bar{J}_\mu(\theta)), \quad \text{where } J_\mu(\theta) := \theta + \frac{\mu \underline{\theta} \cdot \delta_{\underline{\theta}}(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} [\alpha - \omega(s)] dF(s)}{\alpha f(\theta)}.$$

Moreover, denote

$$\theta^L := \min \left\{ \hat{\theta} \in [\underline{\theta}, \bar{\theta}] : \int_{\hat{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s) \geq 0 \right\}.$$

Let $\mu^* := (\mathbf{E}[\omega] - \alpha)_+$. Then the optimal allocation function is

$$q^*(\theta) = \begin{cases} q_{\mu^*}(\theta) & \text{for } \theta_L \leq \theta \leq \bar{\theta}, \\ q^{\text{LF}}(\theta) & \text{for } \underline{\theta} \leq \theta < \theta^L. \end{cases}$$

Proof Sketch of Theorem 3: Overview

We prove **Theorem 3** in three steps:

- #1. Apply basic mechanism design tools \rightsquigarrow constrained mechanism design.
- #2. Show that no topping up (IR) constraint binds for lowest type θ .
- #3. Guess and verify optimal Lagrange multipliers for all other (IR) constraints.
Check that the optimal mechanism also satisfies topping up (TU) constraints.

▶ skip ahead

Step #1: Applying Basic Mechanism Design Tools

The social planner maximizes weighted total surplus:

$$\max_{(q,t)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} + \alpha \underbrace{[t(\theta) - cq(\theta)]}_{\text{total profit}} \right] dF(\theta),$$

subject to (IC), (LS), and (IR).

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Claim 1 (Myerson, 1981; Milgrom and Segal, 2002). A mechanism (q, t) satisfies (IC) if and only if q is nondecreasing and

$$\theta v(q(\theta)) - t(\theta) = \underbrace{\underline{\theta} v(q(\underline{\theta})) - t(\underline{\theta})}_{=U} + \int_{\underline{\theta}}^{\theta} v(q(s)) ds,$$

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subject to (IR), with the change of variables $v(\theta) = v(q(\theta))$.

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Step #2: No Topping Up and Binding (IR) Constraints

The social planner maximizes weighted total surplus:

$$\max_{\substack{U \leq \theta \nu(\theta), \\ \nu \text{ nondecreasing}}} \left\{ [\mathbf{E}[\omega] - \alpha] \underline{U} + \int_{\underline{\theta}}^{\bar{\theta}} \left[\left[\alpha \theta - \frac{\int_{\underline{\theta}}^{\bar{\theta}} [\alpha - \omega(s)] dF(s)}{f(\theta)} \right] \nu(\theta) - \alpha c \nu^{-1}(\nu(\theta)) \right] dF(\theta) \right\},$$

subject to

$$\underbrace{\underline{U} + \int_{\underline{\theta}}^{\theta} \nu(s) ds}_{=\theta \nu(\theta) - t(\theta)} \geq \underbrace{\underline{U}^{\text{LF}} + \int_{\underline{\theta}}^{\theta} \nu^{\text{LF}}(s) ds}_{=U^{\text{LF}}(\theta)}, \quad \forall \theta \in [\underline{\theta}, \bar{\theta}]. \quad (\text{IR})$$

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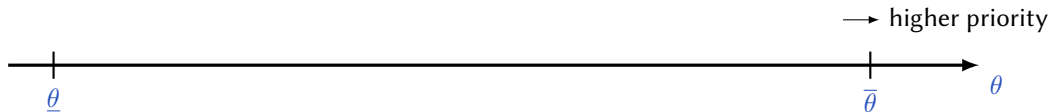
Interpretation: If (IR) binds for some open interval of types (which implies that $\nu = \nu^{\text{LF}}$), then those consumers consume in private market; we say that private market is “active.”

Step #2: No Topping Up and Binding (IR) Constraints

Lemma 1. If the private market is active, then the (IR) constraint binds for the lowest type $\underline{\theta}$.

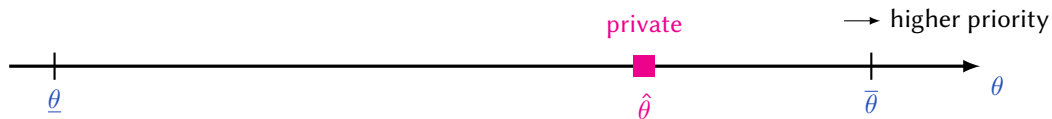
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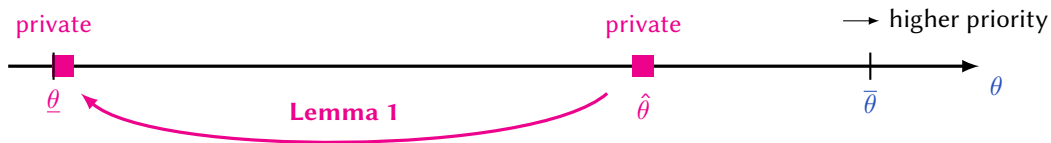
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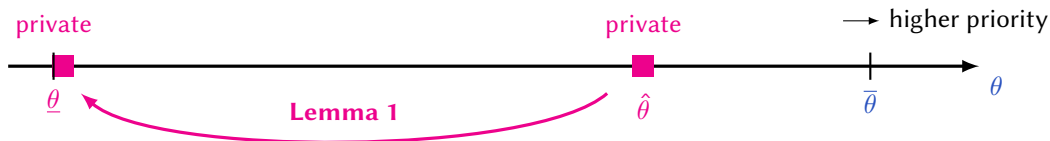
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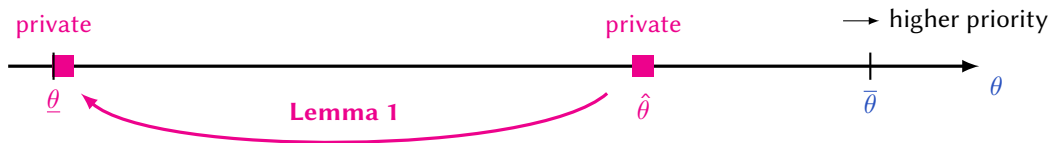
Lemma 1. If the private market is active, then the (IR) constraint binds for the lowest type $\underline{\theta}$.



Intuition: The social planner has no incentive to subsidize the lowest priority consumer $\underline{\theta}$ if she already does not subsidize a positive measure of relatively higher priority consumers.

Step #2: No Topping Up and Binding (IR) Constraints

Lemma 1. If the private market is active, then the (IR) constraint binds for the lowest type $\underline{\theta}$.



Intuition: The social planner has no incentive to subsidize the lowest priority consumer $\underline{\theta}$ if she already does not subsidize a positive measure of relatively higher priority consumers.

Lemma 2. If the private market is inactive, then the (IR) constraint might bind only for $\underline{\theta}$.

Step #2: No Topping Up and Binding (IR) Constraints

The social planner maximizes weighted total surplus:

$$\max_{\substack{U \leq \underline{\theta} \nu(\underline{\theta}), \\ \nu \text{ nondecreasing}}} \left\{ [\mathbf{E}[\omega] - \alpha] \underline{U} + \int_{\underline{\theta}}^{\bar{\theta}} \left[\left[\alpha \theta - \frac{\int_{\underline{\theta}}^{\theta} [\alpha - \omega(s)] dF(s)}{f(\theta)} \right] \nu(\theta) - \alpha c \nu^{-1}(\nu(\theta)) \right] dF(\theta) \right\},$$

subject to

$$\underbrace{\underline{U} + \int_{\underline{\theta}}^{\theta} \nu(s) ds}_{=\theta \nu(\theta) - t(\theta)} \geq \underbrace{\underline{U}^{\text{LF}} + \int_{\underline{\theta}}^{\theta} \nu^{\text{LF}}(s) ds}_{=U^{\text{LF}}(\theta)}, \quad \forall \theta \in [\underline{\theta}, \bar{\theta}]. \quad (\text{IR})$$

Given that (IR) binds for lowest type $\underline{\theta}$ (cf. **Lemmas 1** and **2**), $\underline{U} = \underline{U}^{\text{LF}}$.

Step #2: No Topping Up and Binding (IR) Constraints

The social planner maximizes weighted total surplus:

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subject to a **majorization** (second-order stochastic dominance) constraint,

$$\int_{\underline{\theta}}^{\theta} \nu(s) ds \geq \int_{\underline{\theta}}^{\theta} \nu^{\text{LF}}(s) ds, \quad \forall \theta \in [\underline{\theta}, \bar{\theta}]. \quad (\text{IR})$$


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If ν is affine, then solution is an extreme point (Kleiner et al., 2021; Akbarpour  al., 2024).


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If ν is affine, then solution is an extreme point (Kleiner et al., 2021; Akbarpour  al., 2024).

Technical difficulty: ν is strictly concave, so solution is generally an interior point.

Step #3: Guessing Lagrange Multipliers for (IR) Constraints

To overcome this technical difficulty, we adapt approach of [Amador and Bagwell \(2013\)](#):

- #1.** Guess a continuum of optimal Lagrange multipliers for (IR) constraints.
- #2.** Solve for optimal mechanism and verify that (IR) constraints are satisfied.
- #3.** Verify that complementary slackness conditions are satisfied.

Step #3: Guessing Lagrange Multipliers for (IR) Constraints

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- #1. Guess a continuum of optimal Lagrange multipliers for (IR) constraints.
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- #3. Verify that complementary slackness conditions are satisfied.

This allows us to obtain the optimal mechanism with no topping up (**Theorem 3**).

With positive correlation, optimal mechanism also satisfies topping up (IR) constraints.

[With negative correlation, repeat the above approach with topping up (IR) constraints.]

Negative Correlation

Optimal Mechanisms: Negative Correlation

Theorem 4(a). When correlation is negative (i.e., ω is decreasing), topping up **does not affect** the optimal subsidy program if $\alpha \geq \max \omega$ or $\alpha \leq \min \omega$.

Intuition: If $\alpha \leq \min \omega$, distort consumption \uparrow for all consumers.



Optimal Mechanisms: Negative Correlation

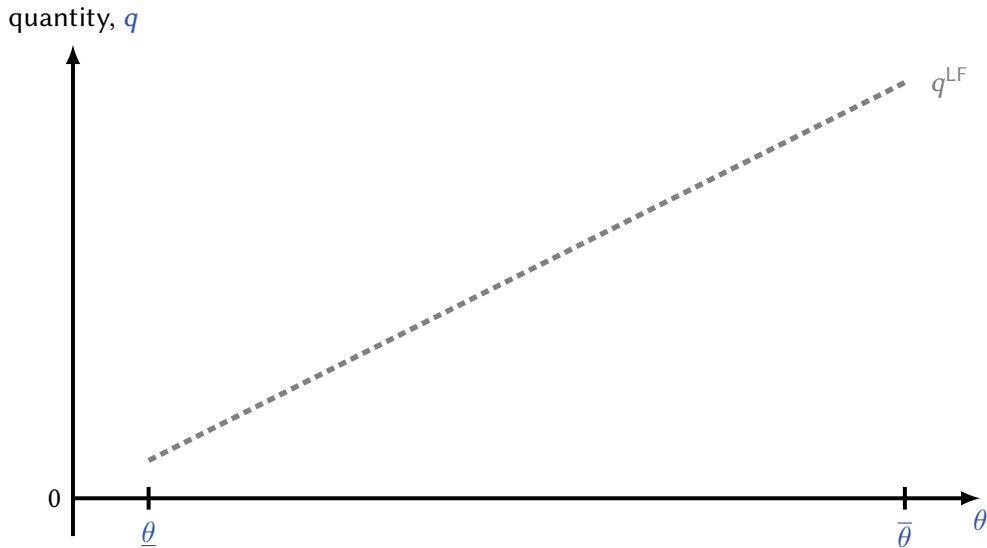
Theorem 4(a). When correlation is negative (i.e., ω is decreasing), topping up **does not affect** the optimal subsidy program if $\alpha \geq \max \omega$ or $\alpha \leq \min \omega$.

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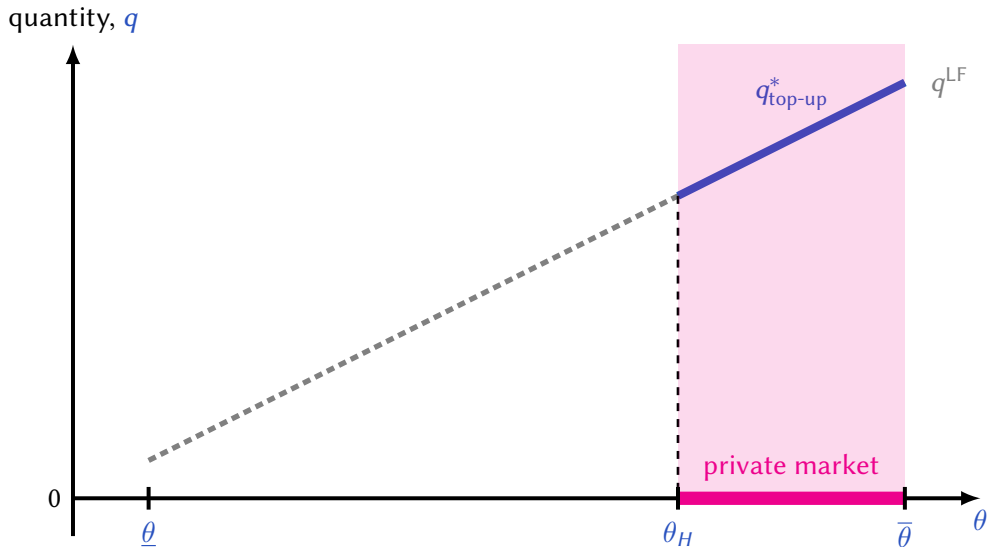


Theorem 4(b). When correlation is negative, there exists $\alpha_{\min} \in [\min \omega, \mathbf{E}[\omega])$ such that topping up **does affect** the optimal subsidy program iff $\alpha_{\min} < \alpha < \max \omega$.

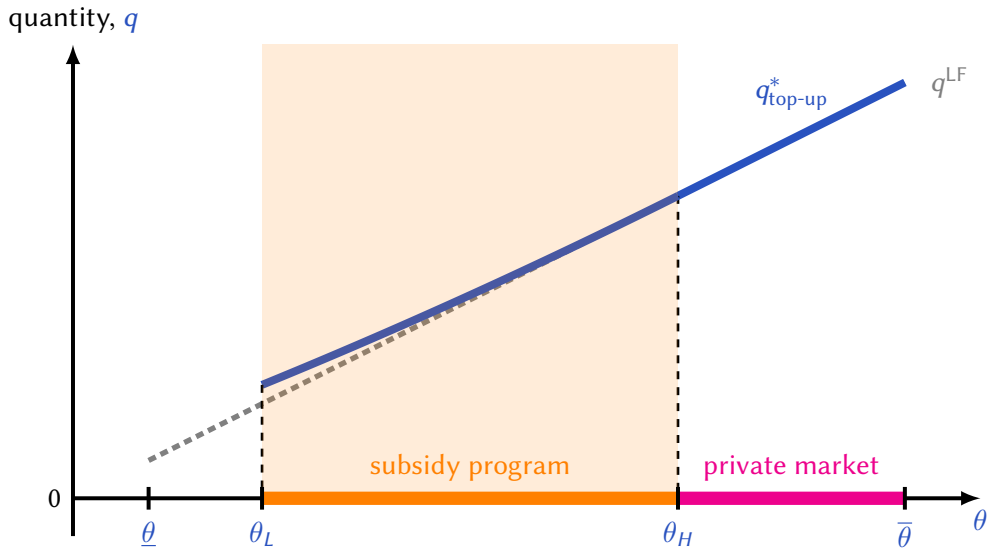
Characterization of Optimal Mechanisms: $\alpha_{\min} < \alpha < \mathbf{E}[\omega]$



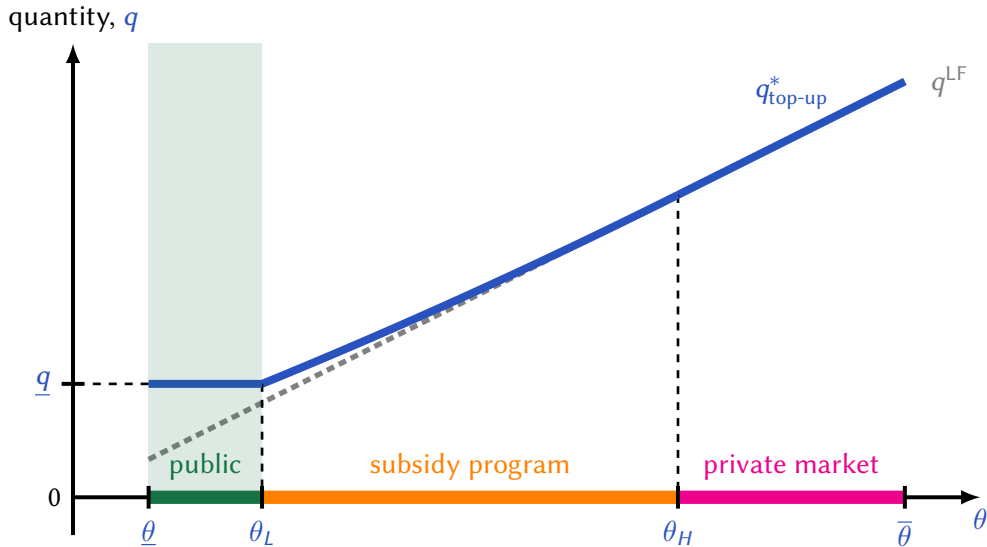
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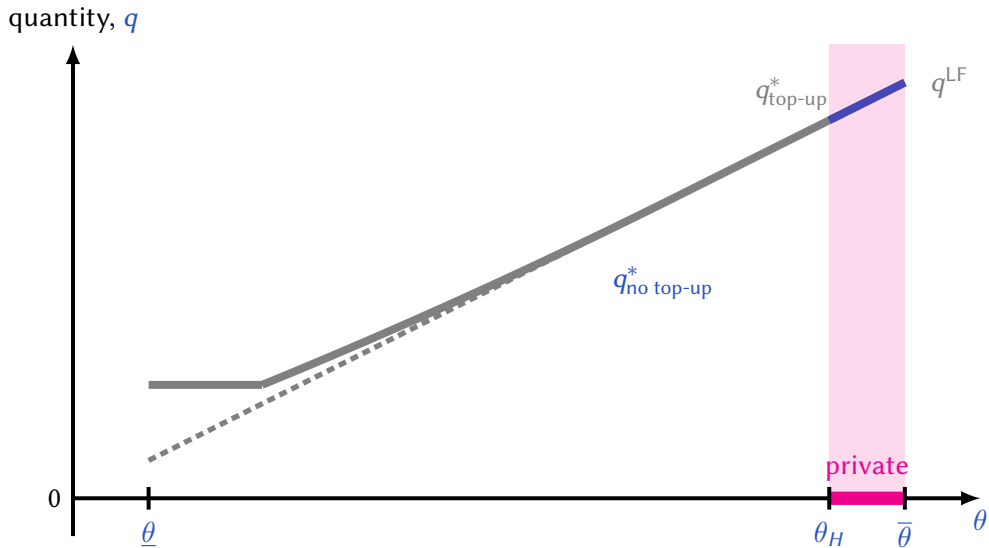
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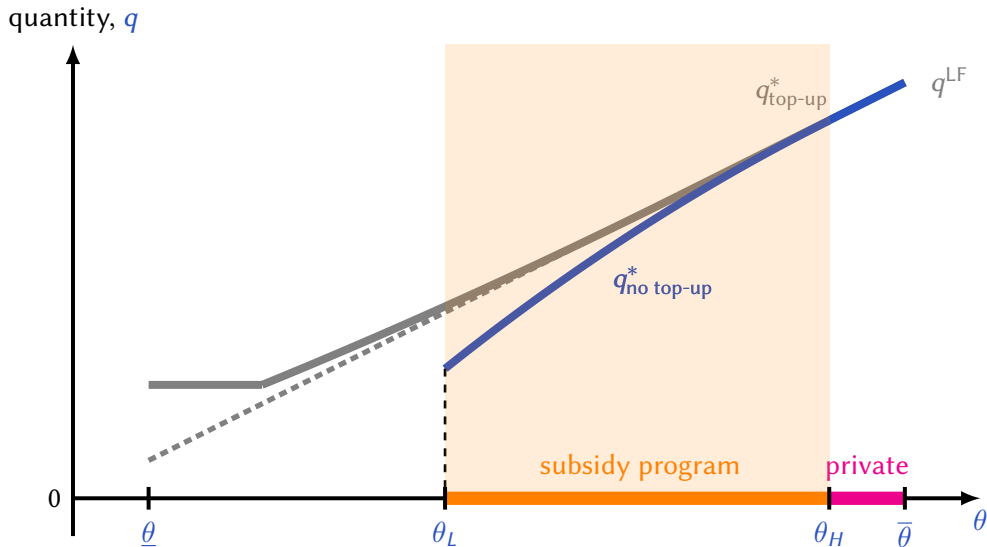
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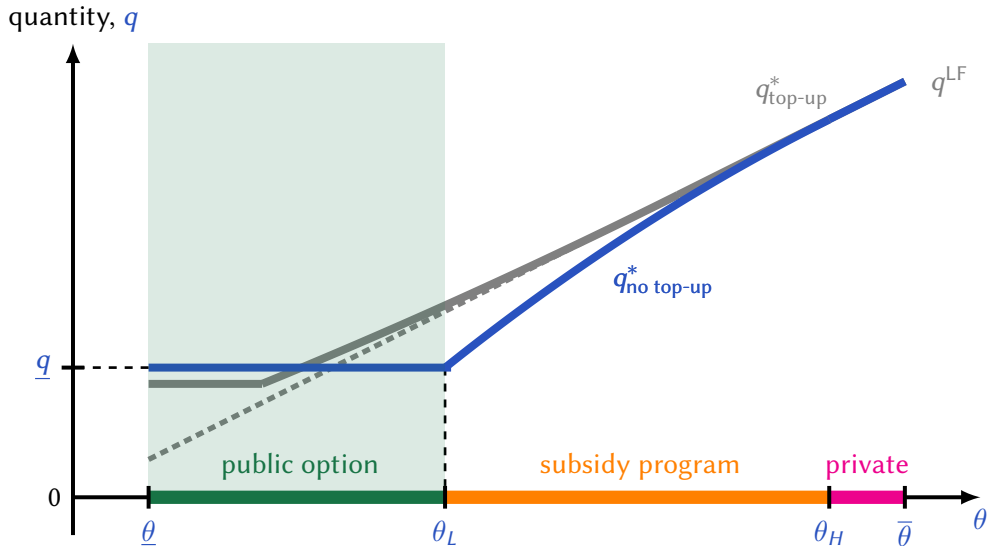
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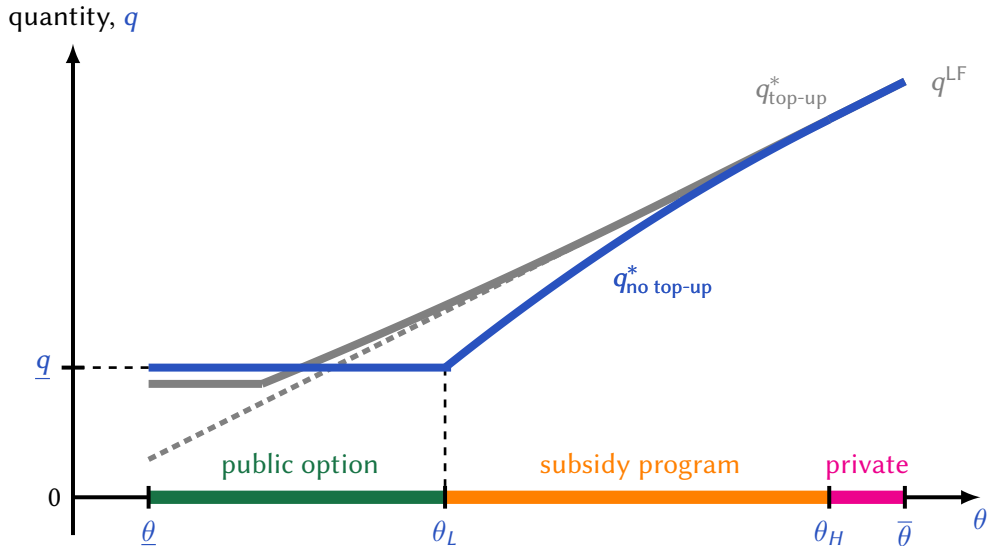
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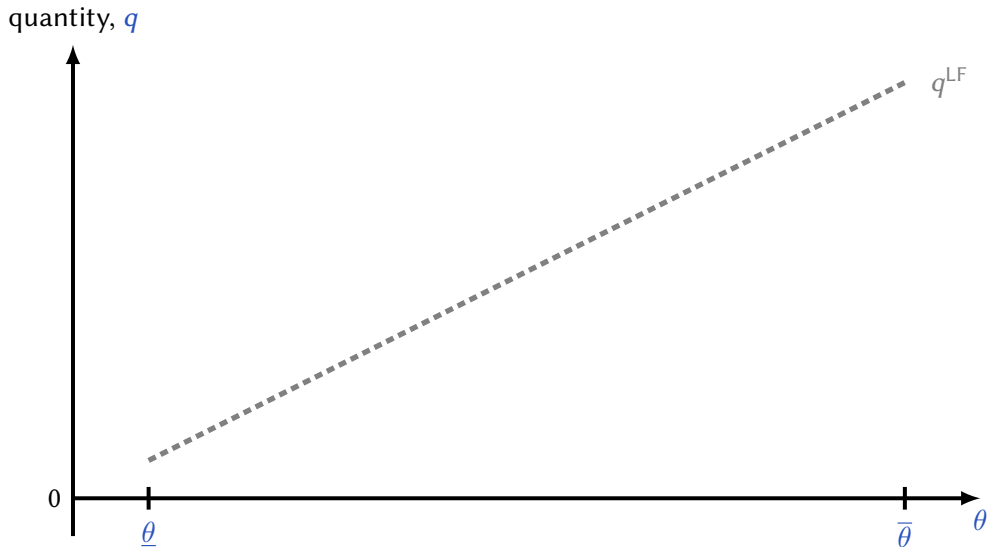
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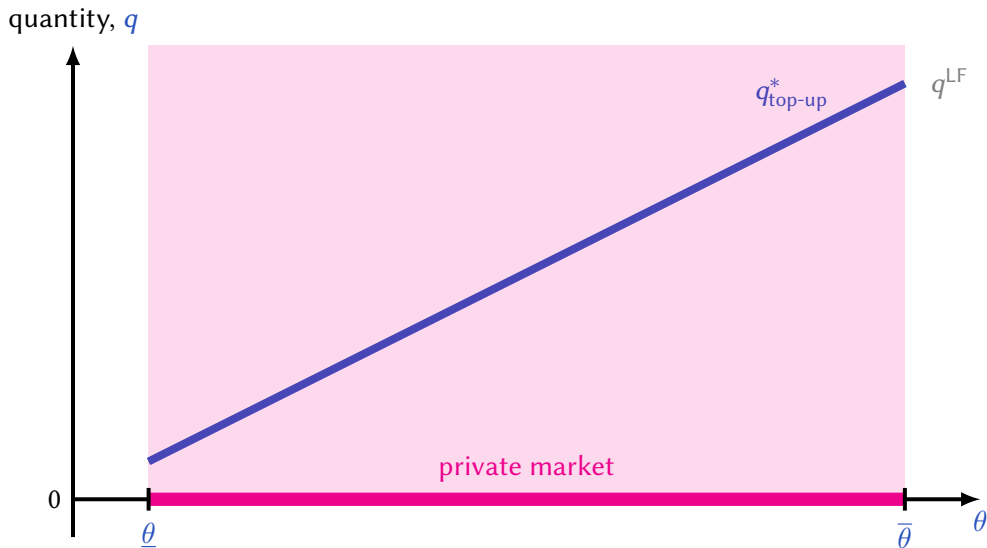
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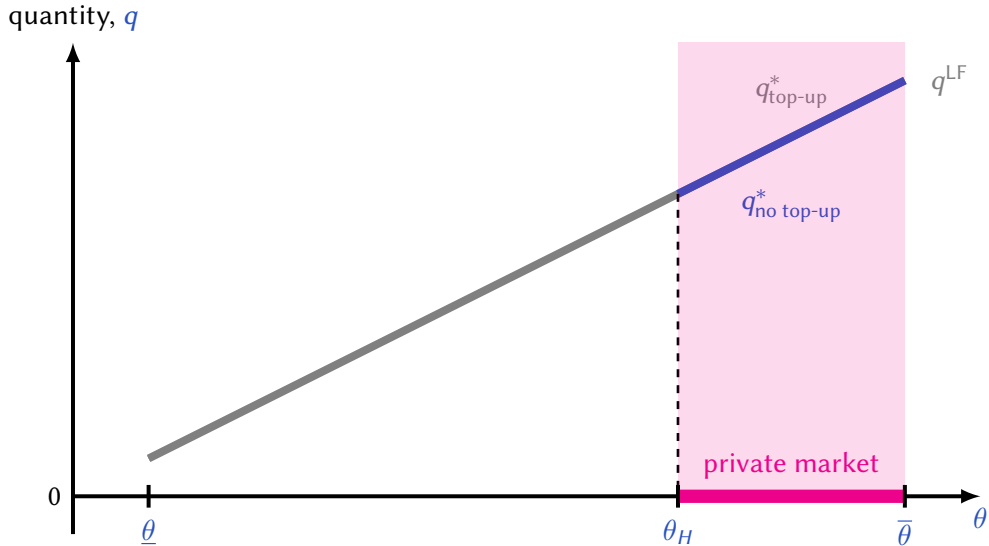
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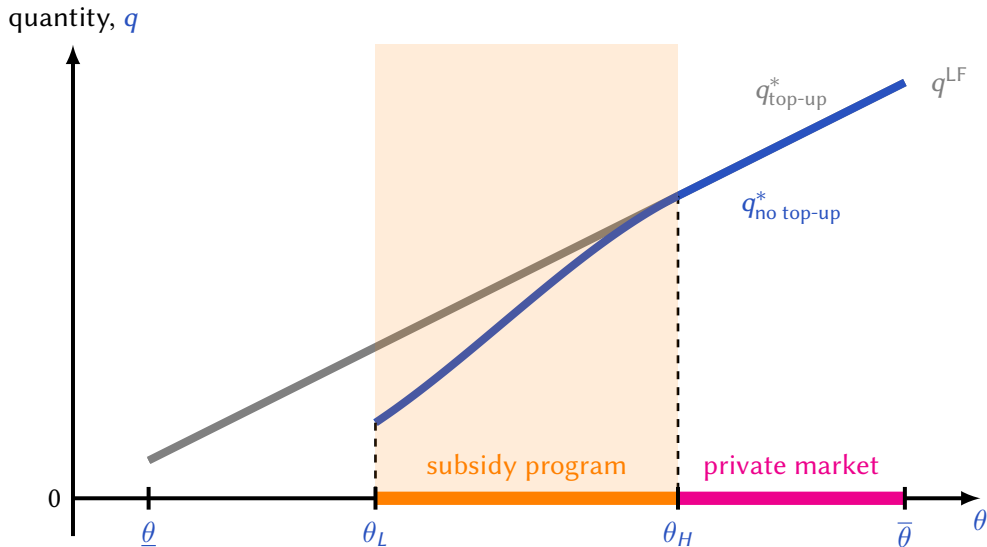
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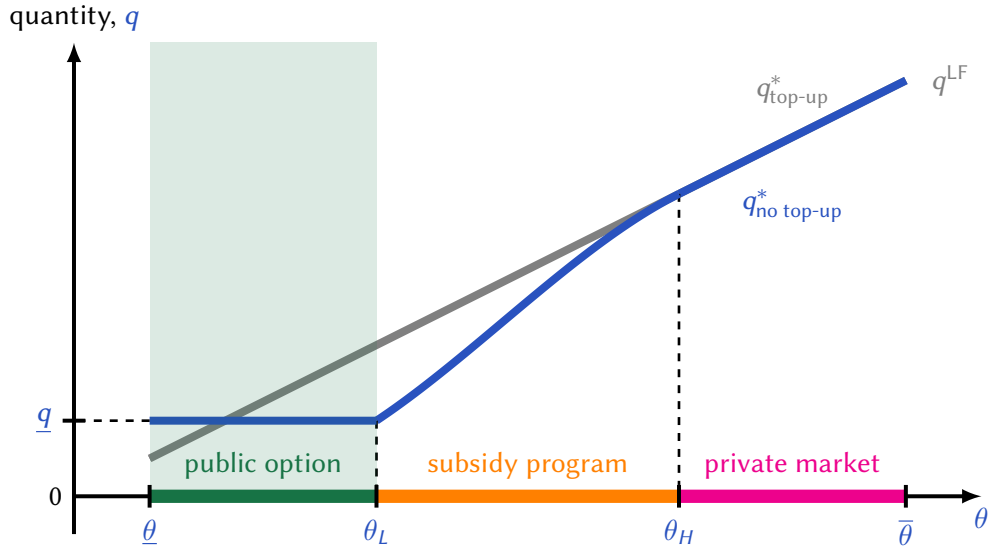
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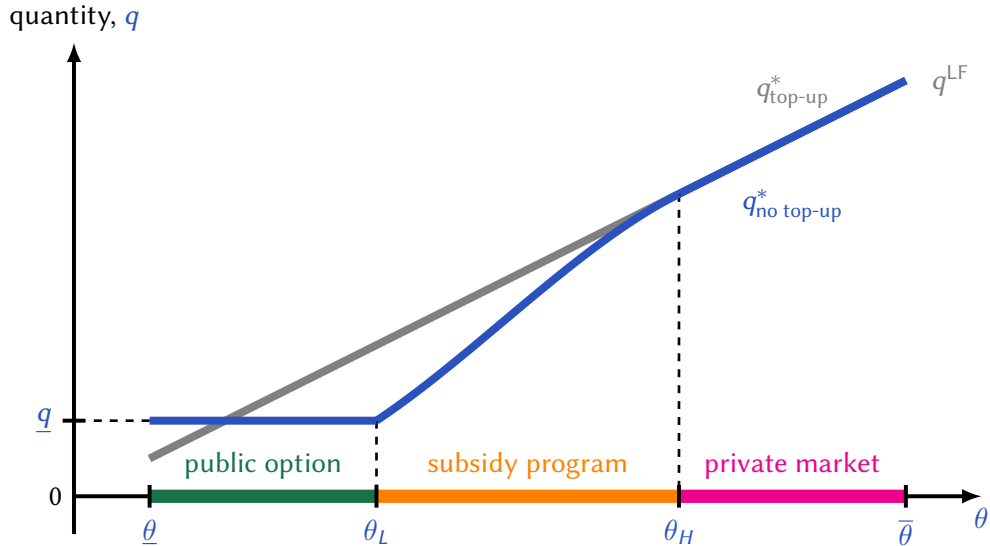
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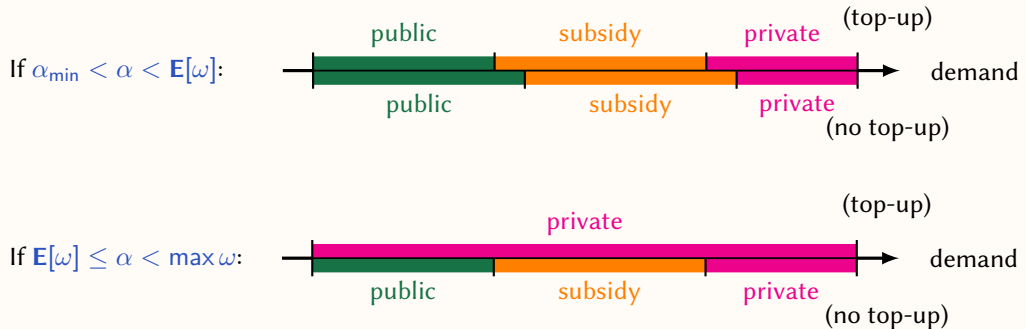


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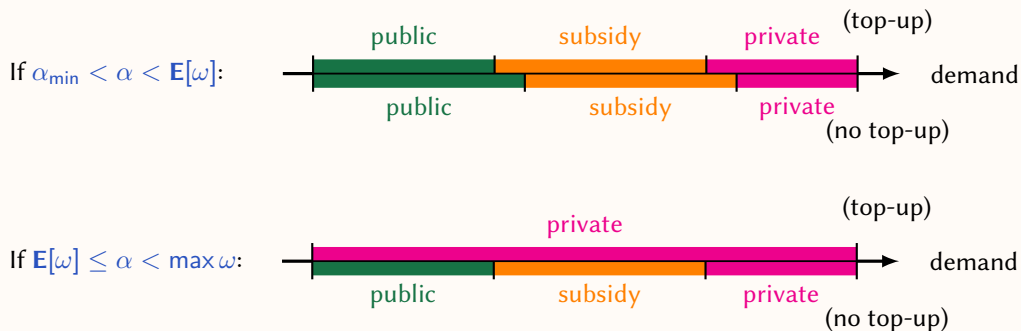
Optimal Mechanisms: Negative Correlation

Theorem 4. When correlation is negative (i.e., ω is decreasing), there exists $\alpha_{\min} \in [\min \omega, \mathbf{E}[\omega]]$ such that topping up **affects** optimal redistribution if and only if $\alpha_{\min} < \alpha < \max \omega$.



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Topping up expands the use of the private market and reduces the use of a free public option.

Discussion

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#1. When should we redistribute in kind?

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- **This paper:** iff $\max_{\hat{\theta} \in [\underline{\theta}, \bar{\theta}]} \mathbf{E} \left[\omega(\theta) \mid \theta \geq \hat{\theta} \right] > \alpha$ or $\max \omega > \alpha$, because of inability to tax.

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#2. When should we use a free public option?

- **Full design:** when $\mathbf{E}[\omega] > \alpha$ (cf. Proposition 1 of Akbarpour  al., 2024).
- **This paper:** same if topping up is allowed, more (even if $\mathbf{E}[\omega] < \alpha$) if not.

Conclusion

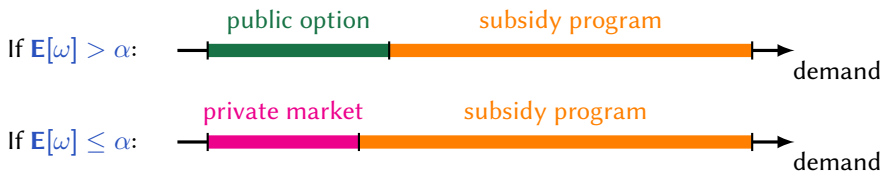
“Programs with or without the possibility of topping up have different welfare properties. Currently, there are no general results regarding the merits of the two. [...] Nor are there any general results on the characterization of optimal public provision policies, targeted or universal.”

—Currie and Gahvari (2008)

So, what have we learned about how topping up affects optimal redistribution?

This Paper

- ① Topping up reduces the social planner's scope for screening.
- ② Topping up can distort screening even conditional on intervention.
- ③ When correlation is positive, optimal mechanisms coincide:



- ④ When correlation is negative, optimal mechanisms differ iff $\alpha \in (\alpha_{\min}, \max \omega)$:
 - ① Topping up rotates consumption away from high priority consumers.
 - ② Topping up expands the use of the private market.
 - ③ Topping up reduces the use of a free public option.

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