Topping Up and Optimal Subsidies

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Motivation

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   - public housing programs (e.g., public housing developments, LIHTC).

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Topping up can be a design choice of subsidy programs:

- Implementation through consumer vouchers or institutional subsidies?
- Explicit banning? E.g., housing loan subsidies may depend on apartment size.

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This paper: How does topping up affect the optimal design of subsidy programs?

$\rightsquigarrow$ **Subsidies $\neq$ money**: we restrict social planner's ability to use lump-sum transfers.

This Paper

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Main Results:

#1. characterization of how topping up impacts the scope for subsidies; and

#2. characterization of how topping up impacts the optimal subsidy program.

Model

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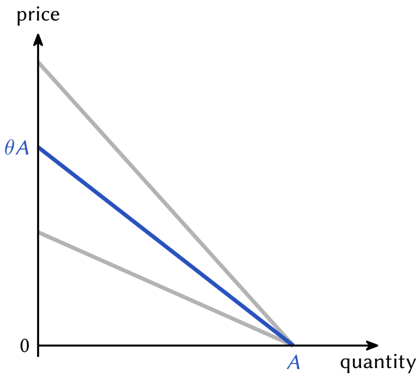
- Each consumer derives utility $\theta v(q) - t$ from quantity $q \in [0, A]$ at total price t.

  Assume that $v : [0, A] \to \mathbb{R}$ is increasing and strictly concave (*i.e.*, $v' > 0$ and $v'' < 0$).

  Example: $v(q) = Aq - \frac{1}{2}q^2$ (linear demand).

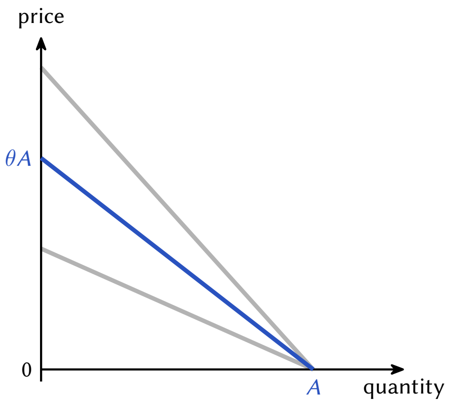


Laissez-Faire Equilibrium

- Since the private market is perfectly competitive, price = marginal cost = c.
- Each consumer solves

$$U^{\mathsf{LF}}(\theta) := \max_{q\in[0,A]} \left[\theta v(q) - cq\right].$$

Since v is strictly concave, there is a unique maximizer, which we denote by $q^{\mathsf{LF}}(\theta)$.



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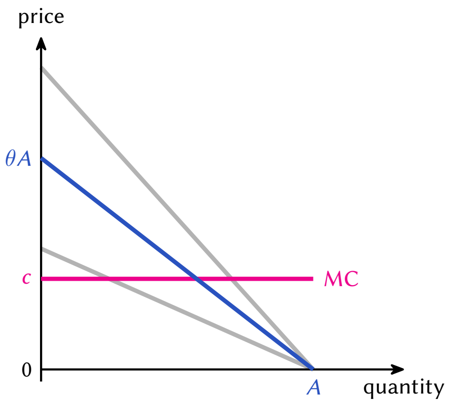

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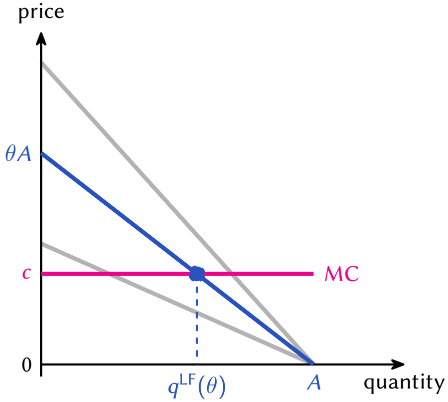

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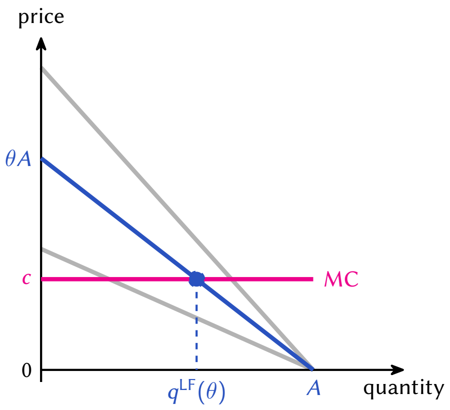

- Assumption of constant MC can be relaxed.
  - Upward-sloping supply: Kang (2023).
    Eqm. effects $\Rightarrow$ alt. redistribution channel.
  - **This paper:** participation constraints.

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  - the allocation function $q : [\underline{\theta}, \overline{\theta}] \to [0, A]$ denoting *total* consumption by consumer;
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- We focus on **(in-kind) subsidies**: $t(\cdot) \geq 0 \quad \rightsquigarrow$ no lump-sum transfers to consumers.

  However, we allow for lump-sum transfers to consumers "outside of the mechanism."

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  - $\rightsquigarrow$ $\omega(\theta)$: expected social value of giving him one unit of money.
  - $\rightsquigarrow$ Intuitively, monotonicity of ω captures correlation between consumption and social value.

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For any given mechanism (q, t), the weighted total surplus can be written as:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} + \alpha \underbrace{[t(\theta) - cq(\theta)]}_{\text{total profit}} \right] \mathrm{d}F(\theta).$$

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subject to

- incentive compatibility, $$\theta \in \operatorname*{arg\,max}_{\hat{\theta} \in [\underline{\theta}, \overline{\theta}]} \left[\theta v(q(\hat{\theta})) - t(\hat{\theta}) \right], \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]; \qquad \text{(IC)}$$
- no lump-sum transfers, $$t(\theta) \geq 0, \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]; \qquad \text{(LS)}$$
- individual rationality,

$$\begin{cases} \text{topping up:} & q(\theta) \geq q^{\mathsf{LF}}(\theta), \\ \text{no topping up:} & \theta v(q(\theta)) - t(\theta) \geq U^{\mathsf{LF}}(\theta), \end{cases} \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]. \qquad \text{(IR)}$$

Related Work

- **Redistributive Mechanism Design.** Che, Gale and Kim (2013); Condorelli (2013); Dworczak ⓡ Kominers ⓡ Akbarpour (2021); Akbarpour ⓡ Dworczak ⓡ Kominers (2022); Akbarpour ⓡ Dworczak ⓡ Kominers (2024); Pai and Strack (2024); Vairo (2025).
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- **"Partial" Mechanism Design.** Philippon and Skreta (2012); Tirole (2012); Fuchs and Skrzypacz (2015); Dworczak (2020); Loertscher and Muir (2022); Kang and Muir (2022); Kang (2023).
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- **Methodological Tools in Mechanism Design.**
  - Generalized ironing: Toikka (2011).
  - Lagrangian approach: Amador, Werning and Angeletos (2006); Amador and Bagwell (2013).
  - Type-dependent outside options: Jullien (2000), Dworczak and Muir (2024).
  - Majorization constraints (in a concave program): Kleiner, Moldovanu and Strack (2021).
  - ⇝ **This paper:** majorization constraints due to type-dependent outside options (in a convex program).

Related Work

- **Public Finance.**

  Theory: Nichols and Zeckhauser (1982), Blackorby and Donaldson (1988), Coate (1989), Besley and Coate (1991), Coate, Johnson and Zeckhauser (1994), Gahvari and Mattos (2007).

  Empirics: Baum-Snow and Marion (2009), Diamond and McQuade (2019), van Dijk (2019), Dinerstein, Neilson and Otero (2020), Atal, Cuesta, González and Otero (2021), Dinerstein and Smith (2021), Jiménez-Hernández and Seira (2021), Handbury and Moshary (2021).

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  – As Doligalski ⓡ Dworczak ⓡ Akbarpour ⓡ Kominers (2025) write:

    "*Several decades after the original work of Atkinson and Stiglitz (1976), conventional economic wisdom seems to have embraced the idea of using income taxes rather than goods market interventions to redistribute; it would appear that this intuition needs to be revisited, and more research is needed to understand whether it constitutes good policy advice under realistic scenarios.*"

Main Results

Theorem 1. The optimal mechanism strictly improves on the laissez-faire outcome iff:

$$\begin{cases} \max\limits_{\hat{\theta} \in [\underline{\theta}, \overline{\theta}]} \mathbf{E}\left[\omega(\theta) \mid \theta \geq \hat{\theta}\right] > \alpha, & \text{if topping up is allowed,} \\ \max \omega > \alpha, & \text{if topping up is not allowed.} \end{cases}$$

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Topping up strictly reduces scope for subsidies unless correlation is partly positive: $\omega(\overline{\theta}) = \max \omega$.

Optimal Mechanisms: Positive Correlation

Theorem 2. When correlation is positive (i.e., ω is increasing), topping up **does not affect** the optimal subsidy program: the optimal mechanisms with and without topping up coincide.

Examples: childcare, disability care, public transit, basic staples like coarse bread/cassava.

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The *Public Expenditure Handbook* published by the IMF recommends that:

> "[m]arketed goods with a negative income elasticity (i.e., inferior goods) are ideal candidates for a redistributive subsidy."

Theorem 2 complements the "self-targeting" explanation: no need to prevent topping up.

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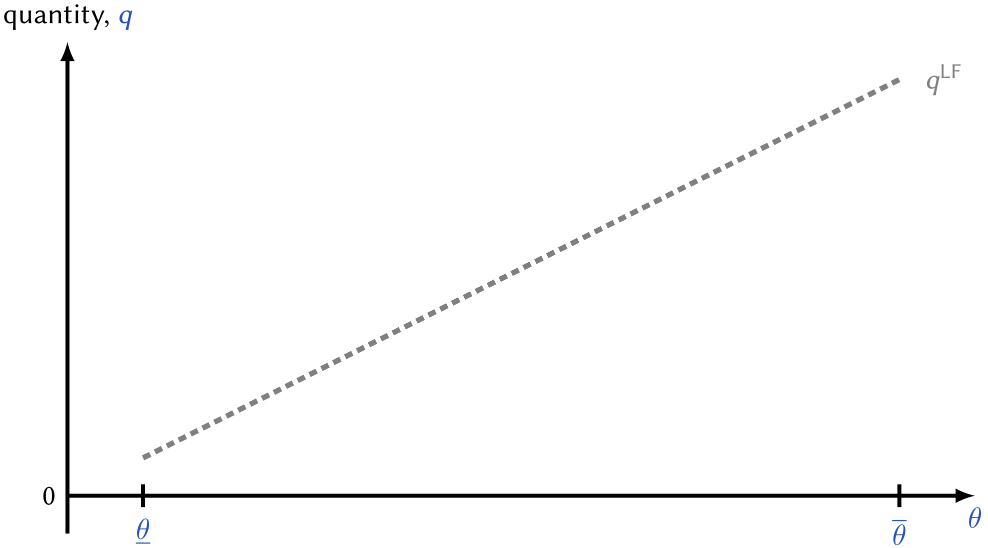

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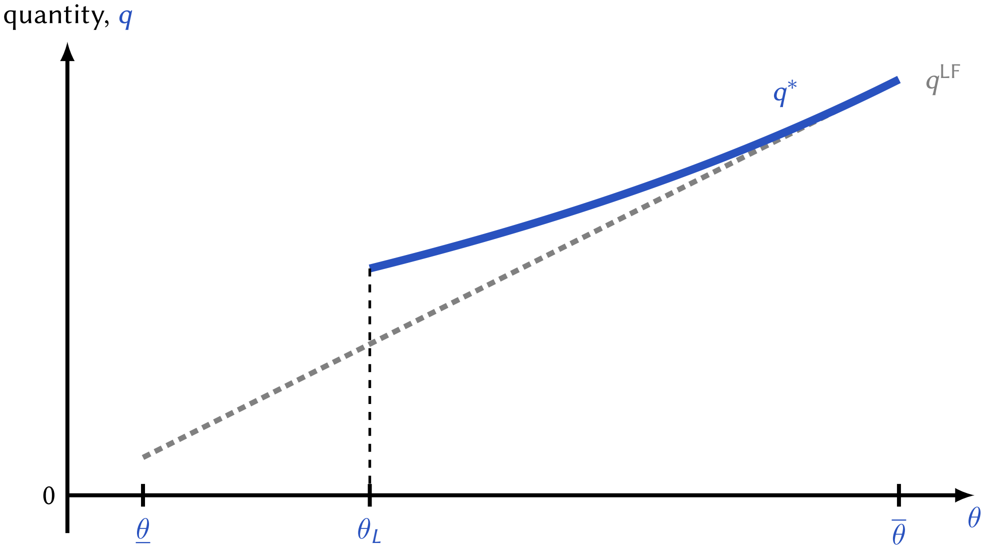

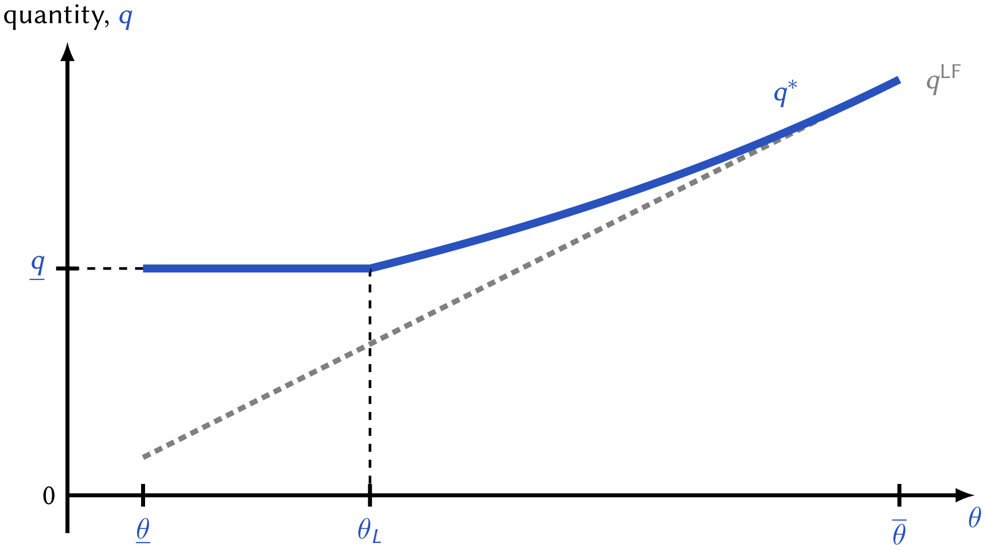
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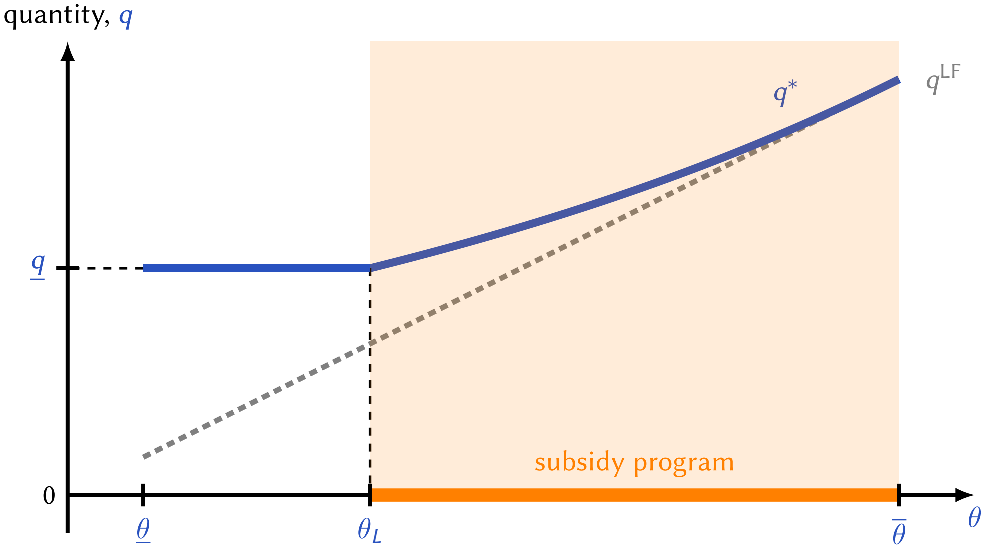

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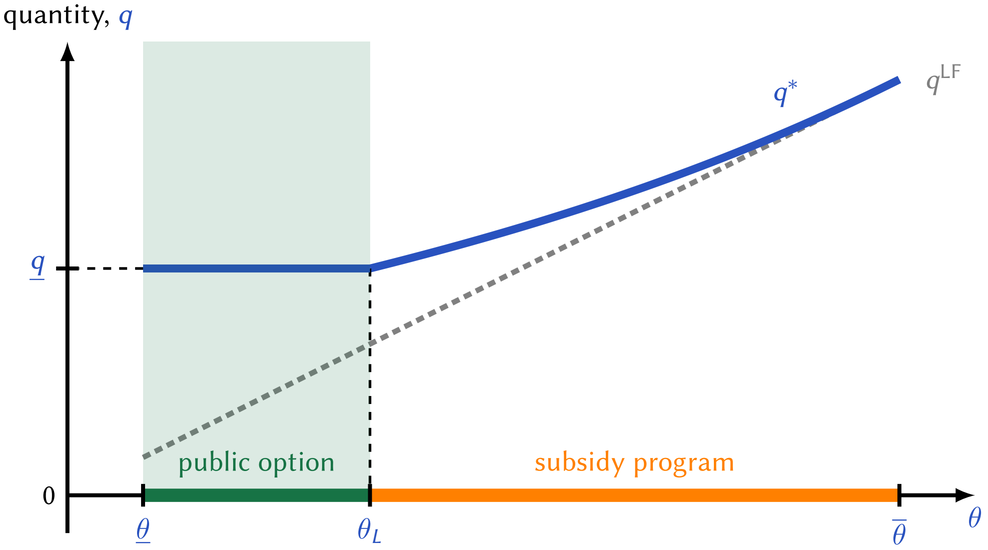

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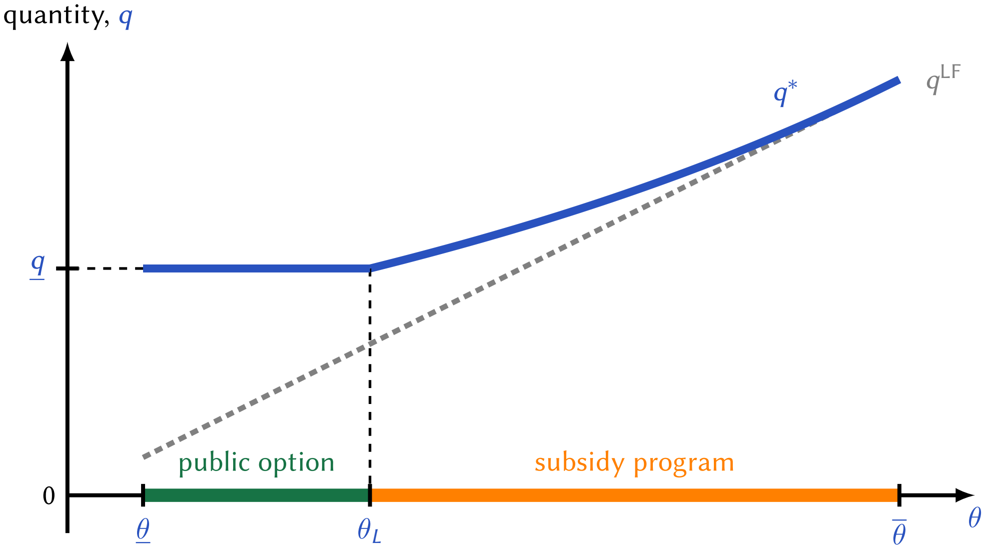

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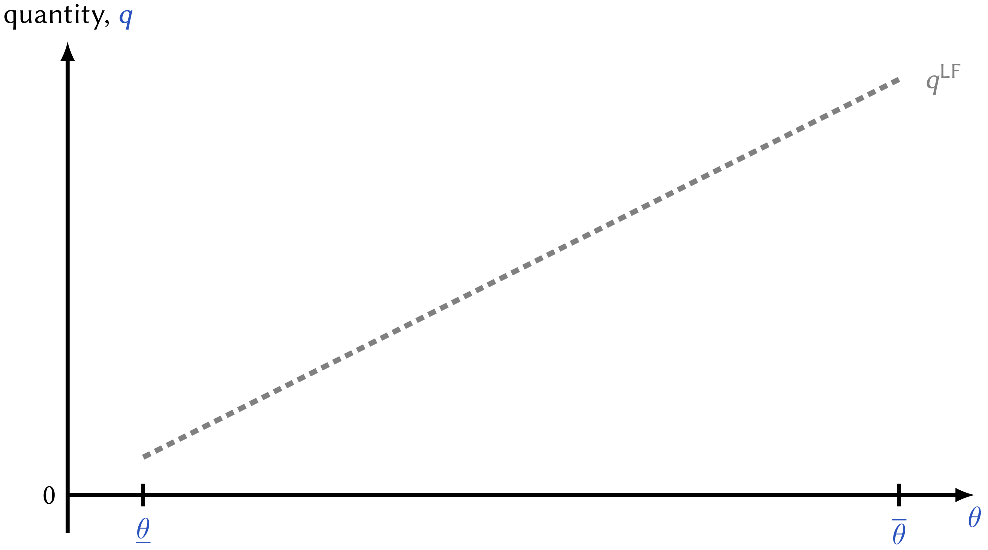

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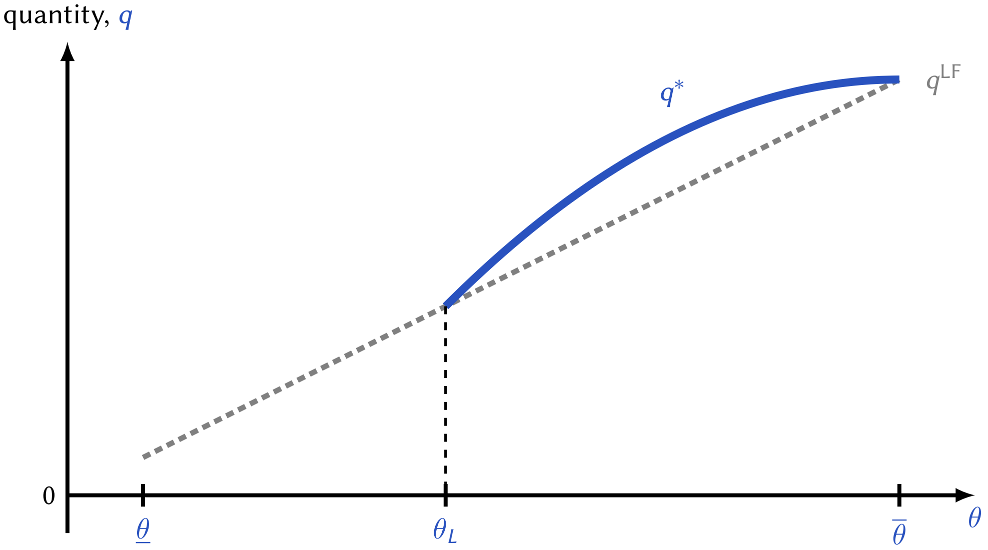

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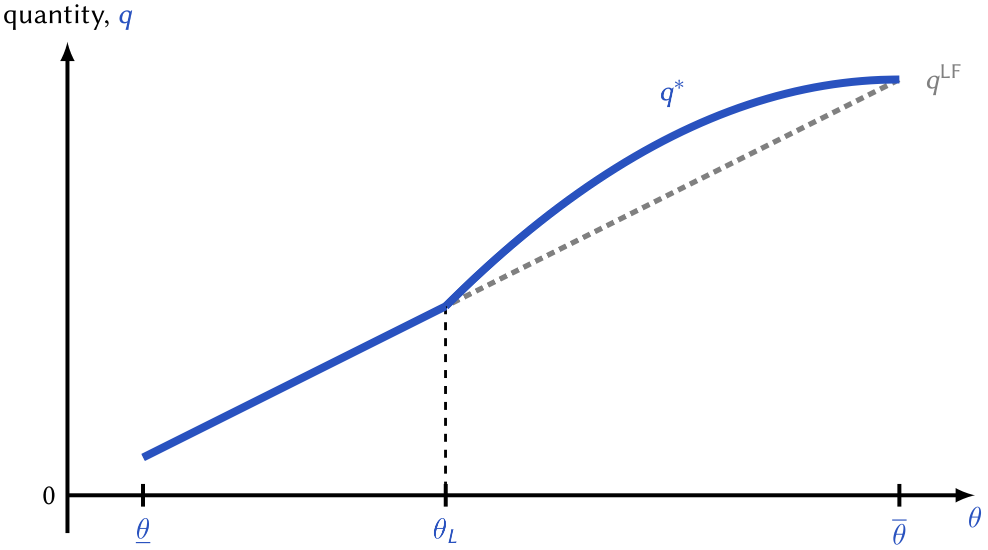

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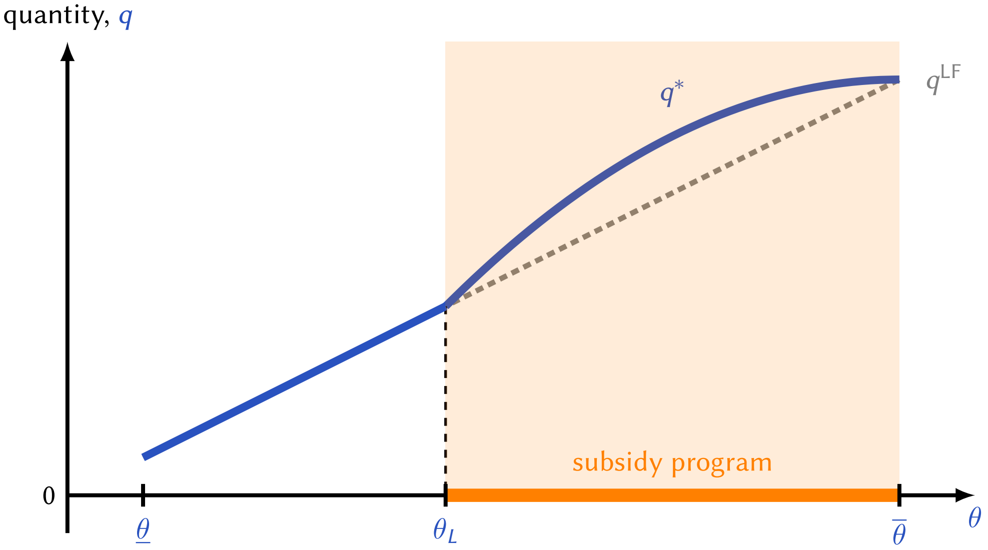

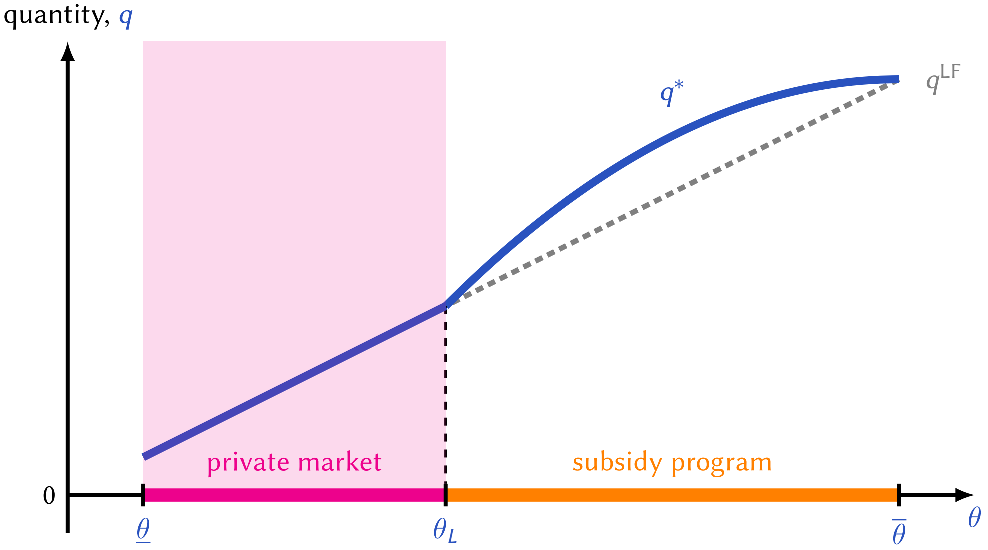
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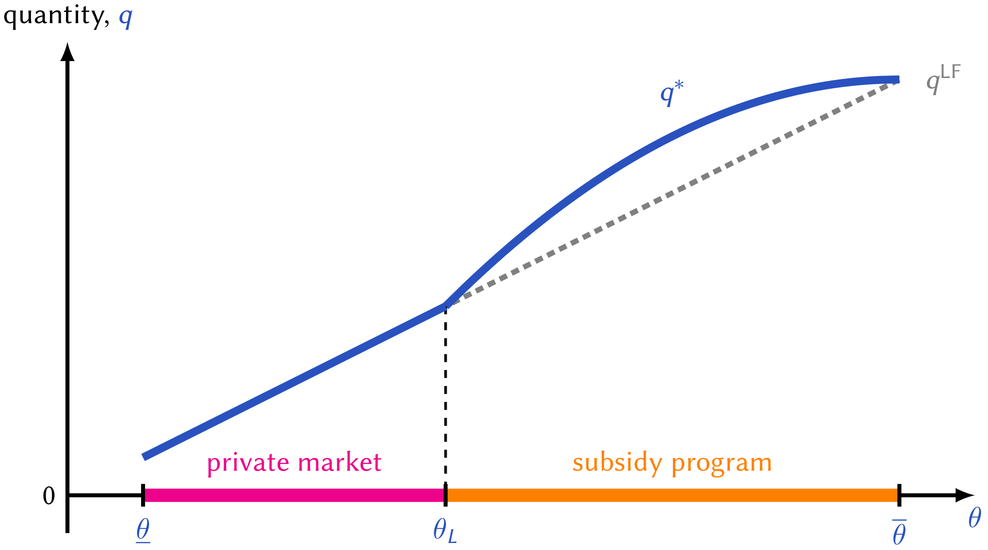

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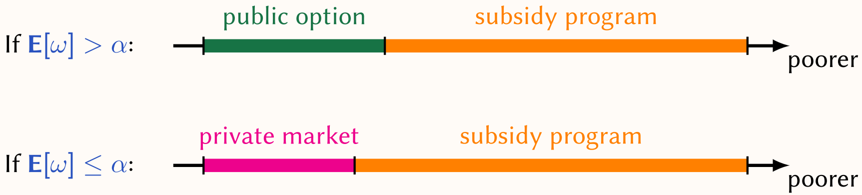

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In particular, for any $\mu \geq 0$, define

$$q_\mu(\theta) := D\left(c, \overline{J_\mu}(\theta)\right), \quad \text{where } J_\mu(\theta) := \theta + \frac{\mu\underline{\theta} \cdot \delta_{\underline{\theta}}(\theta) - \int_\theta^{\overline{\theta}} [\alpha - \omega(s)] \, \mathrm{d}F(s)}{\alpha f(\theta)}.$$

Moreover, denote

$$\theta_L := \min\left\{\theta \in [\underline{\theta}, \overline{\theta}] : \int_\theta^{\overline{\theta}} [\alpha - \omega(s)] \, \mathrm{d}F(s) \leq 0\right\}.$$

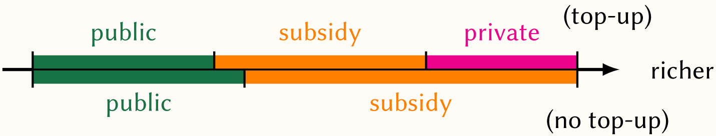
Let $\mu^* := (\mathbf{E}[\omega] - \alpha)_+$. Then the optimal allocation function is

$$q^*(\theta) = \begin{cases} q_{\mu^*}(\theta) & \text{for } \theta_L \leq \theta \leq \overline{\theta}, \\ q^{\mathrm{LF}}(\theta) & \text{for } \underline{\theta} \leq \theta < \theta_L. \end{cases}$$

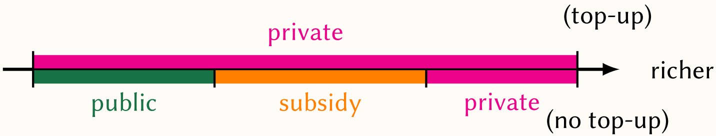
Optimal Mechanisms: Negative Correlation

Theorem 3. When correlation is negative (i.e., ω is decreasing), topping up **does not affect** the optimal subsidy program if $\min \omega \geq \alpha$. However,

If $\mathbf{E}[\omega] > \alpha > \min \omega$:



If $\mathbf{E}[\omega] \leq \alpha < \max \omega$:



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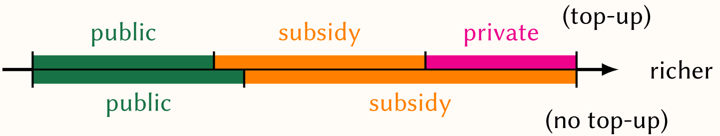

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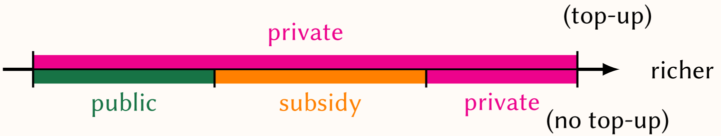

Topping up expands the use of the private market and reduces the use of a free public option.

Conclusion

> "Programs with or without the possibility of topping up have different welfare properties. Currently, there are no general results regarding the merits of the two. [...] Nor are there any general results on the characterization of optimal public provision policies, targeted or universal."
>
> —Currie and Gahvari (2008)

So, what have we learned about how topping up affects the design of optimal subsidies?

This Paper

1. Topping up strictly reduces the scope of subsidies, unless correlation is partly positive.

2. When correlation is positive, the optimal subsidy programs coincide:

If $\mathbf{E}[\omega] > \alpha$:

If $\mathbf{E}[\omega] \leq \alpha$:

3. When correlation is negative, the optimal subsidy programs coincide if $\min \omega \geq \alpha$.

   If $\min \omega < \alpha < \max \omega$, then:

   a. Topping up expands the use of the private market.
   b. Topping up reduces the use of a free public option.

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